Numerical study of quantum turbulence in superfluid helium

HVBK two-fluid model

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ANR project QUTE-HPC: QUantum Turbulence Exploration by High-Performance Computing

Agence Nationale de la Recherche

ANR Project QUTE-HPC (2019-2023)

10 members = 5 Physics + 5 Mathematics

- (HPC) parallel codes for QT :: open source,
- huge simulations of physical configurations (compare with our own experiments).

http://qute-hpc.math.cnrs.fr/

Overview

1. Two-fluid HVBK model and quantum turbulence
   - Two-fluid conception of superfluid helium
   - Quantum turbulence and vortex lines
   - The HVBK model and the mutual friction force

2. Results of direct simulation of HVBK
   - Hydrodynamic behaviors accounting for the mutual friction
   - The role of the friction force on the energy budget
   - The role of the friction force on the turbulence intermittency

Two-fluid conception of superfluid helium

Liquide Helium under 2.1K viscous paradox: frictionless but viscous
@Alfred Leitner 1963 from Michigan stat university

- The superfluid:
  no viscosity
  free of entropy

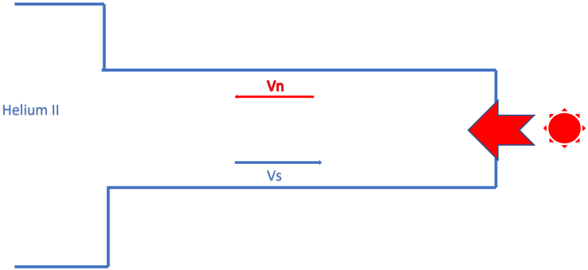
- The normal fluid:
  viscosity
  carries entropy

helium II two-fluids model (Landau 1941)

- Normal fluid: ρ_n, v_n, ν_n
- Superfluid: ρ_s, v_s
- For $0K < T < T_\lambda$ mixture of two fluid $\rho = \rho_n + \rho_s$, for $T > T_\lambda$, $\rho_s/\rho = 0$; for $T = 0K$, $\rho_n/\rho = 0$

Thermal counter-flow and quantum turbulence

Second sound wave oscillation of T and S, first sound wave ocsillation of ρ and P



$$Q = \rho S T v_n, \quad j = \rho_s v_s + \rho_n v_n = 0 \text{ and } V_{ns} = v_n - v_s = Q/(ST\rho_s)$$

Heat transfer in liquid helium like a sound wave

"Second sound" $u_2^2 = TS^2 \rho_s / C\rho_n$

$Q > Q_{critical}$ then vortex line are created

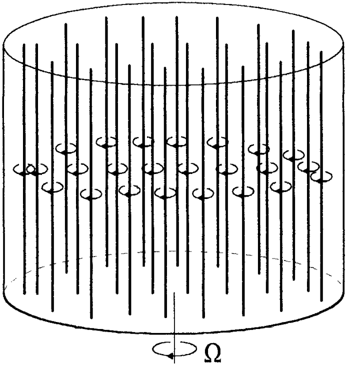
Quantum turbulence contains a large number of randomly distributed vortex lines

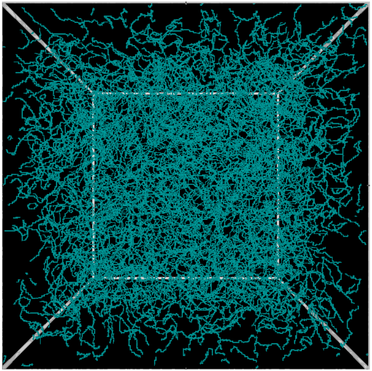
Vortex lines in helium II

- The circulation is quantized in unit circulation of $\frac{\hbar}{m}$, with $\hbar$ is the Plank's constant and m is the helium atom mass, thus

$$\kappa = \oint \mathbf{v}_s dr = \frac{\hbar}{m}, \quad \omega = 2\Omega = n\kappa$$

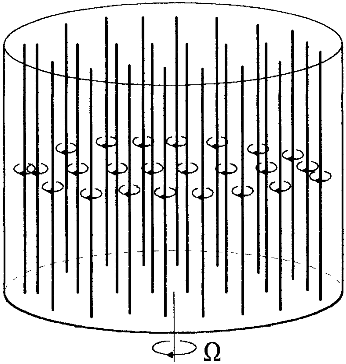
with ω is the voritcity of rotating solide body and n is the volume line density





Vortex lines and mutual friction force

The experiment of second sound wave attenuation propagating in rotating helium II [Vinen & Hall 1956].



$$\frac{A}{A_0} = (1 + \frac{2\Omega}{76.1})^{-1}$$

Mutual friction force

- A volume force: independent to the geometric nor the position
- depends on the relative orientation of relative velocity and the rotating axis
- depends on the angular velocity Ω. Feynman's rule: $2\Omega = \nabla \times v_s = \kappa \mathcal{L}$.
- Model: Each vortex line is the scattering center of the excitation (rotons), where brings the momentum exchange (Newton's Laws)

HVBK two-fluids model

The mutual friction force writes:

$$\boldsymbol{F}_M = -B\frac{\rho_s\rho_n}{\rho}\frac{\boldsymbol{\omega}\wedge(\boldsymbol{\omega}\wedge(\boldsymbol{v}_s-\boldsymbol{v}_n))}{|\boldsymbol{\omega}|} - B'\frac{\rho_s\rho_n}{\rho}\boldsymbol{\omega}\wedge(\boldsymbol{v}_s-\boldsymbol{v}_n) \tag{1}$$

with B and $B^{'}$ are two temperature dependent parameter, and $\omega = \nabla\times\boldsymbol{v}_s$, $\boldsymbol{v}_n$ is the normal fluid velocity and $\boldsymbol{v}_s$ is the superfluid velocity.
The governing equations write:

$$\nabla.\boldsymbol{v}_n = 0,\ \nabla.\boldsymbol{v}_s = 0 \tag{2}$$

$$\frac{\partial\boldsymbol{v}_n}{\partial t} + \nabla.(\boldsymbol{v}_n\otimes\boldsymbol{v}_n) = -\nabla p_n + \frac{1}{\rho_n}\boldsymbol{F}_M + \nu_n\Delta\boldsymbol{v}_n + \boldsymbol{f}_{ext} \tag{3}$$

$$\frac{\partial\boldsymbol{v}_s}{\partial t} + \nabla.(\boldsymbol{v}_s\otimes\boldsymbol{v}_s) = -\nabla p_s - \frac{1}{\rho_s}\boldsymbol{F}_M + \nu_s\Delta\boldsymbol{v}_s + \boldsymbol{f}_{ext} \tag{4}$$

Vortex lines polarized? turbulence $\rightarrow$ random vortex lines, bundle coherent structure

Good estimation on macroscopic scales

3DT HVBK solver

Code HVBK 3d turbulence: p3dfft(2D decomp), pseudo-spectral method, 3D periodic boundary conditions. Parallel computing on cluster Myria CRIANN.

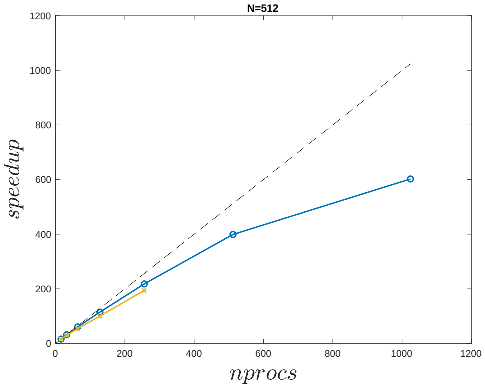


Figure: The scaling test N=512 of code (○) 2D decomposition (x)1D decomposition, Speedup = Time(nprocs = 1)/Time(nprocs = N).

• The slop ”-5/3” on inertial scales of energy spectrum, dissipation scale decreases with temperature decreases, the coupling energy loss due to the relative velocity.

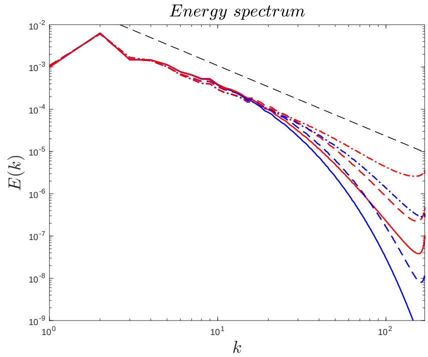


Figure: (a) Turbulence energy spectrum for different temperature (density ratios); (b) Spectrum for dissipation, relative velocity and mutual friction force energy.

- Two fluid are strongly correlated with each other, the energy exchanging is strongly associate to the vorticity.

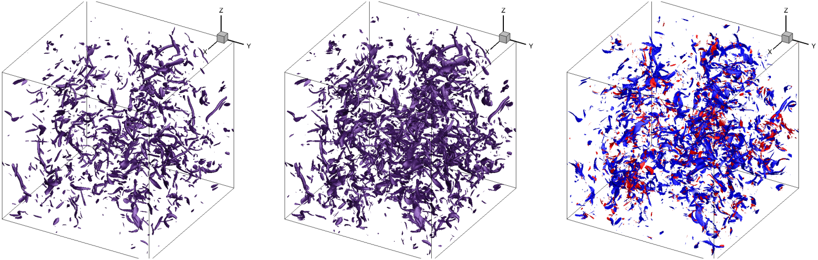


Figure: (a) Snapshot of iso-contour $\omega_n^2 = 5rms(\omega_s^2)$ in normal fluid, (b) Snapshot of $\omega_s^2 = 5rms(\omega_s^2)$ in super fluid, (c) Energy exchange *un.Fns* in normal fluid. (blue) iso-contour of 6rms, (red) iso-contour of -3rms.

Direct simulation of HVBK

- The energy exchange mainly occurs in the intense vorticity region and it is intermittent.

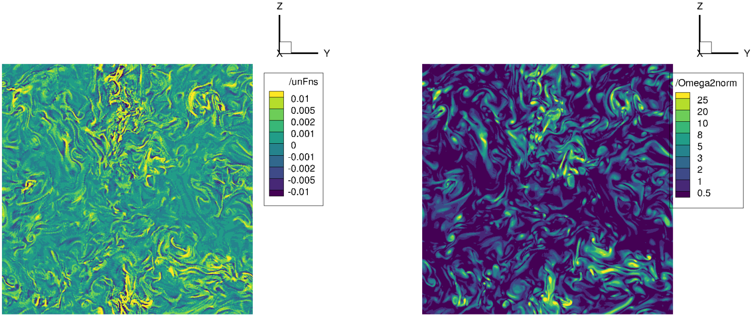


Figure: (left)The energy exchange in normal fluid $u_n.F_{ns}$, (right) the enstrophy of normal fluid $|\Omega_n|^2$

Direct simulation of HVBK

• The dissipation rate ε follows the log-normal PDF according to the eddy-cascade Richardson model as well as the coupling energy PDF.

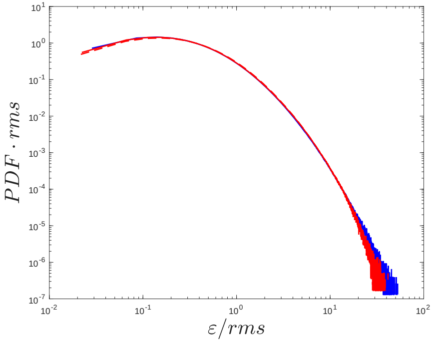


Figure: (left) The normalised P.D.F of locale viscous energy transfers rate, the form is similar to a log-normal distribution (well known as the classical turbulent). (right) The normalised P.D.F of locale energy exchanging rate (red) ϕ_n and (blue) ϕ_s, (- -) a log-normal distribution with $\mu = -\frac{\sigma^2}{2}$ and $\sigma^2 = 0.6931$.

We follow the classical approach in HIT for the second order structure function transport equations, and integration from 0 to r obtain the 4/3 law, viz.

$$-\overline{\delta u_{\parallel}^{n}(\delta u_i^n)^2} - \frac{1}{r^2}\int_0^r s^2\mathcal{L}^n ds + 2\nu_n \frac{d}{dr}\overline{(\delta u_i^n)^2} = \frac{4}{3}\overline{\epsilon}^n r, \quad (5)$$

$$-\overline{\delta u_{\parallel}^{s}(\delta u_i^s)^2} - \frac{1}{r^2}\int_0^r s^2\mathcal{L}^s ds + 2\nu_s \frac{d}{dr}\overline{(\delta u_i^s)^2} = \frac{4}{3}\overline{\epsilon}^s r, \quad (6)$$

with

$$\mathcal{L}^n \equiv -2\frac{\rho_s}{\rho}\overline{(\delta u_i^n)(\delta F_i^{ns})} - 2\overline{(\delta u_i^n)(\delta f_i^n)}, \quad (7)$$

$$\mathcal{L}^s \equiv 2\frac{\rho_n}{\rho}\overline{(\delta u_i^s)(\delta F_i^{ns})} - 2\overline{(\delta u_i^s)(\delta f_i^s)}, \quad (8)$$

where $u_{\parallel}$ is the velocity component parallel to vector $\vec{r}$, $r = |\vec{r}|$, $\delta f = f(\vec{x}+\vec{r}) - f(\vec{x})$ and for isotropic δf depends only r

The scales-by-scales energy budget

$$-\overline{\delta u_{\parallel}(\delta u_i)^2} - \frac{1}{r^2}\int_0^r s^2 \mathcal{L} ds + 2\nu \frac{d}{dr}\overline{(\delta u_i)^2} = \frac{4}{3}\bar{\epsilon} r, \tag{9}$$

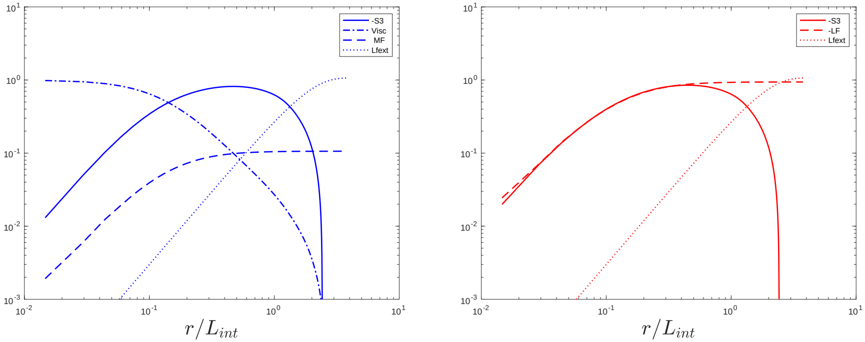


Figure: The case $\rho_n/\rho = 0.9$ and $\nu_s = 0$ (–) The convection term $-\overline{\delta u_{\parallel}(\delta u_i)^2}$, (- -) the mutual friction term $2\frac{\rho_s}{\rho}\frac{1}{r^2}\int_0^r s^2\overline{(\delta u_i^n)(\delta F_i^{ns})}ds$ for normal fluid and $2\frac{\rho_n}{\rho}\frac{1}{r^2}\int_0^r s^2\overline{(\delta u_i^s)(\delta F_i^{ns})}ds$ (– •) the viscous term (•••) the external forcing term. Normalised by $4/3\varepsilon^n$

The scales-by-scales energy budget

$$-\overline{\delta u_{\parallel}(\delta u_i)^2} - \frac{1}{r^2}\int_0^r s^2 \mathcal{L} ds + 2\nu \frac{d}{dr}\overline{(\delta u_i)^2} = \frac{4}{3}\bar{\epsilon} r, \tag{10}$$

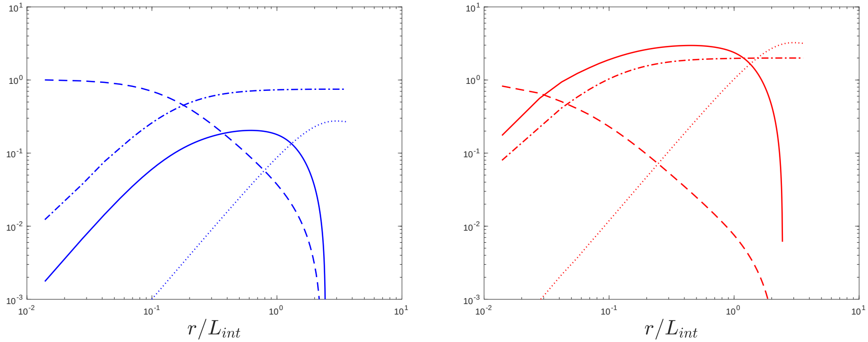


Figure: The case $\rho_n/\rho = 0.1$ and $\nu_s = 0.1\nu_n$ (–) The convection term $-\overline{\delta u_{\parallel}(\delta u_i)^2}$, (- -) the mutual friction term $2\frac{\rho_s}{\rho}\frac{1}{r^2}\int_0^r s^2\overline{(\delta u_i^n)(\delta F_i^{ns})}ds$ for normal fluid and $2\frac{\rho_n}{\rho}\frac{1}{r^2}\int_0^r s^2\overline{(\delta u_i^s)(\delta F_i^{ns})}ds$ (– •) the viscous term (•••) the external forcing term. Normalised by $4/3\varepsilon^n$ for the normal fluid and Normalised by $4/3\varepsilon^s$ for the superfluid.

$$\partial_t D_{111} + \left(\partial_r + \frac{2}{r}\right) D_{1111} - \frac{6}{r} D_{1122} = -T_{111} + 2\nu C - 2\nu Z_{111} + MF + Fext, \qquad (11)$$

with $\partial_r \equiv \partial/\partial r$,

$$\begin{aligned} D_{111} &= \overline{(\delta u)^3}; \\ D_{1111} &= \overline{(\delta u)^4}; \\ D_{1122} &= \overline{(\delta u)^2(\delta v)^2}; \\ C(r,t) &= -\frac{4}{r^2} D_{111}(r,t) + \frac{4}{r}\partial_r D_{111} + \partial_r\partial_r D_{111}; \\ Z_{111} &= \overline{3\delta u \left[\left(\frac{\partial u}{\partial x_l}\right)^2 + \left(\frac{\partial u'}{\partial x'_l}\right)^2\right]}, \end{aligned} \qquad (12)$$

where $\delta u = u(x+r) - u(x)$ is the longitudinal velocity increment, $\delta v = v(x+r) - v(x)$ is the transverse velocity increment and double indices indicate summation and a prime denotes variables at point $x + r$. Finally,

$$T_{111} = \overline{3(\delta u)^2 \, \delta\left(\frac{\partial p}{\partial x}\right)}. \qquad (13)$$

The vortex stretching and the intermittency

$$\partial_t D_{111} + \left(\partial_r + \frac{2}{r}\right) D_{1111} - \frac{6}{r} D_{1122} = -T_{111} + 2\nu C - 2\nu Z_{111} + MF + Fext, \quad (14)$$

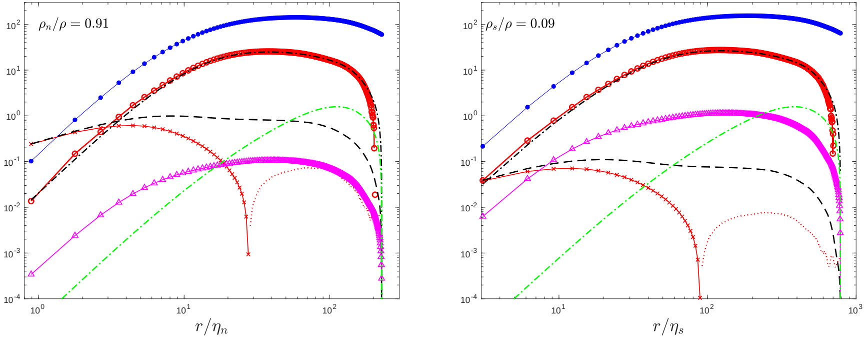


Figure: Balances of different terms in equations for the normal fluid (left) and the superfluid (right) for density ratios $\rho_n/\rho = 0.91$. ($\bullet-$) $(\partial_r + 2/r)D1111$, ($\circ-$) $(\partial_r + 2/r)D1111 - 6/r.D1122$, (black $-\cdot$) $-T111$, ($\times-$) $-2\nu C$ ($\cdots$) positive part of $2\nu C$, (- -) $-2\nu Z111$, ($\triangle-$) coupling terms, (green $-\cdot$) for the external forcing term. The plots are dimensionless.

$$\partial_t D_{111} + \left(\partial_r + \frac{2}{r}\right) D_{1111} - \frac{6}{r} D_{1122} = -T_{111} + 2\nu C - 2\nu Z_{111} + MF + Fext, \qquad (15)$$

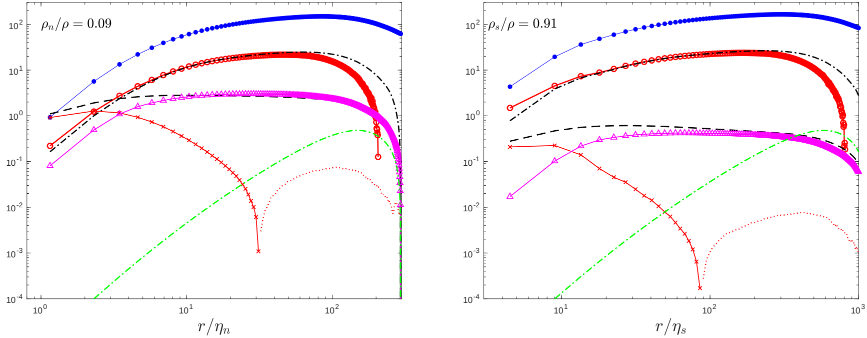


Figure: Balances of different terms in equations for the normal fluid (left) and the superfluid (right) for density ratios $\rho_n/\rho = 0.09$. ($\bullet-$) $(\partial_r + 2/r)D1111$, ($\circ-$) $(\partial_r + 2/r)D1111 - 6/r.D1122$, (black $-\cdot$) $-T111$, ($\times-$) $-2\nu C$ ($\cdot\cdot\cdot$) positive part of $2\nu C$, (- -) $-2\nu Z111$, ($\triangle-$) coupling terms, (green $-\cdot$) for the external forcing term. The plots are dimensionless.

Conclusions

HVBK two-fluid model DNS show that:

• The normal fluid and the superfluid are highly correlated due to the mutual friction in large scales.

• The mutual friction exchange momentum between the two fluid component, averagely it extract energy from the superfluid and add energy to the normal fluid.

• The large energy exchange mainly happens at the strong vorticity region, which is an intermittent effect.

• The mutual friction has a very important impact to the energy balance. The energy cascading in the inertial sub-range are characterised by both the local dissipation rate and the mutual friction energy exchanging.

• The mutual friction has little impact on the vortex stretching, thus having limited influence to the intermittency of the turbulence.

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