Numerical Simulation of thermo-convective flows induced by electric fields in cylindrical annular cavities with dielectric liquids

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Introduction

- Geophysics concern: how to make experiment that simulates the convection in planets interior? Need for a real radial gravity


- **ESA project: GeoFlow (2008-2017):** Experiment in Fluid Science Laboratory of Columbus on ISS by Brandenburg Technology University in Cottbus (LAS, Prof. C. Egbers)

- **ATMOFLOW 2020-2024**
  
  (DRL for LAS in collaboration with CNES for LOMC)

  Investigation of atmospheric flows near the equatorial zone

- Application 1: Generation of thermal convection in a low-gravity environment

- Application 2: Enhancement of heat transfer by electric voltage in aerospace (with reduced weight constraints)
• The pondermotive force in a dielectric fluid of density \( \rho \) and permittivity \( \varepsilon \) under the action of an electric field \( \vec{E} \) is given by the Helmholtz relation (Landau & Lifshitz, EM, T8)

\[
\vec{f} = \rho_e \vec{E} - \frac{1}{2} \vec{E}^2 \nabla \varepsilon + \nabla \left[ \frac{1}{2} \rho \left( \frac{\partial \varepsilon}{\partial \rho} \right)_T \right] \vec{E}^2
\]

Coulomb force  
Dielectrophoretic (DEP) force  
Electrostriction force

• High frequency electric field \( f >> \max(1/\tau_v, 1/\tau_k, 1/\tau_e, 1/\tau_{ion}) \)  
  
  neglect free charges (Coulomb force)

• Electrohydrodynamic Boussinesq approximation: linear variation of the permittivity and density with temperature

\[
\rho(T) = \rho_0 \left[ 1 - \alpha(T - T_0) \right] \quad \text{and} \quad \varepsilon(T) = \varepsilon_1 \left[ 1 - e(T - T_0) \right]
\]
The dielectrophoretic force can be split into a conservative force and buoyancy force:

\[ \vec{f}_{DEP} = -\frac{1}{2} \vec{E}^2 \nabla \varepsilon = \nabla p_e + \delta \rho \vec{g}_e \]

**Electric gravity:**

\[ \vec{g}_e = \frac{1}{\rho_0 \alpha} \nabla \left( \frac{\varepsilon e}{2} \vec{E}^2 \right) \]

where

\[ \rho(T) = \rho_0 \left[ 1 - \alpha(T - T_0) \right] \]

\[ \varepsilon(T) = \varepsilon_1 \left[ 1 - e(T - T_0) \right] \]

Magnitude of the electric gravity \( \sim V^2, \varepsilon_1, e \)
ELECTRIC GRAVITY IN SIMPLE GEOMETRIES

<table>
<thead>
<tr>
<th>Plane Electrode</th>
<th>Cylindrical Electrode</th>
<th>Spherical Electrode</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_e \propto \frac{V^2}{d^3} )</td>
<td>( g_e \propto \frac{V^2}{r^3} )</td>
<td>( g_e \propto \frac{V^2}{r^5} )</td>
</tr>
</tbody>
</table>

The dielectrophoretic force in spherical geometry models the mechanism of convection in planets [GEOFLOW, B. Futterer & C. Egbers, BTU, Cottbus, Germany].
The electric gravity in dielectric fluid between parallel plates is

\[ g_e = \frac{\varepsilon_1 e V_0^2}{\alpha \rho_0 d^3} \]

The electric gravity: \( g_e \sim V_0^2; \quad g_e \sim \frac{1}{d^3} \)

d = 1 \text{cm}, \ \Delta T = 1 \text{K}, \ V_0 = 1 \text{ V}

<table>
<thead>
<tr>
<th>Fluid</th>
<th>(10^{-3} \rho) (kg m(^{-3}))</th>
<th>(10^3 \alpha) (K(^{-1}))</th>
<th>(\varepsilon_r)</th>
<th>(e) (K(^{-1}))</th>
<th>(g_e) (m s(^{-2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acetonitrile</td>
<td>0.777</td>
<td>1.38</td>
<td>36</td>
<td>0.155</td>
<td>7.11</td>
</tr>
<tr>
<td>Nitrobenzene</td>
<td>1.198</td>
<td>0.830</td>
<td>34.9</td>
<td>0.188</td>
<td>10.92</td>
</tr>
<tr>
<td>Acetone</td>
<td>0.785</td>
<td>1.43</td>
<td>19.1</td>
<td>0.086</td>
<td>1.11</td>
</tr>
<tr>
<td>Chlorobenzene</td>
<td>1.101</td>
<td>0.985</td>
<td>5.61</td>
<td>0.0157</td>
<td>0.01</td>
</tr>
<tr>
<td>Silicone oil M5</td>
<td>0.920</td>
<td>1.08</td>
<td>2.7</td>
<td>(1.065 \times 10^{-3})</td>
<td>(2.73 \times 10^{-5})</td>
</tr>
</tbody>
</table>
Thermoelectric convection in cylindrical annular cavities

\[ \eta = \frac{R_1}{R_2} = 0.5, \quad \Gamma = \frac{L_z}{d} = 20 \]

The effect of a thermo-electric body force on the flow of a dielectric liquid with a radial temperature gradient and an alternating electric voltage in cylindrical annular cavities has been studied:

- by linear stability analysis (infinitely long annulus)
- by weakly nonlinear analysis (infinitely long annulus)
- by direct numerical simulations (DNS) for

\[ f \gg \frac{1}{\tau_y}, \frac{1}{\tau_K} \]

Experiments in parabolic flight campaigns funded by CNES & DLR
LSA in a cylindrical annulus: Critical electric Rayleigh number

\[ L_c \]

- is independent on Pr;
- depends sensitively on \( \eta \) and on \( \gamma_e \)

Heat transfer in the thermoelectric convection in microgravity

The 3 velocity components are of the same order of magnitude in contrast with the Couette-Taylor problem where the azimuthal component is dominant.

$$Nu(L; Pr, \eta) = A(Pr, \eta)L^{\beta(Pr, \eta)}$$

Do we observe columnar or helical vortices?
Δ𝑇 = 4𝐾

Δ𝑇 = 3𝐾

Δ𝑇 = 2𝐾

Δ𝑇 = 1𝐾

𝑉_{peak} = 0kV  𝑉_{peak} = 2kV  𝑉_{peak} = 4kV  𝑉_{peak} = 6kV  𝑉_{peak} = 8kV  𝑉_{peak} = 10kV

Experiment in parabolic flight
Experiment in parabolic flight

Spatio-temporal diagram \((t, \varphi)\) of the intensity at the median surface

\(\Delta T = 10K\)

\(V_E = 3298\)

Instantaneous shadowgraph pictures
Critical parameters for $Pr = 10$
$Ga = 1370 ; \delta = 0.1$

Thermal Mode

Column Mode

Electric Mode

Meyer et al. CRME 345 (2017)
Problem Formulation

Governing equations (Electro-hydrodynamic Boussinesq approximation)

\[
\begin{align*}
\nabla \cdot \mathbf{u} &= 0 \\
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\nabla \pi + \nu \nabla^2 \mathbf{u} - \alpha \theta (\mathbf{g} + \mathbf{g}_e) \\
\frac{\partial \theta}{\partial t} + (\mathbf{u} \cdot \nabla) \theta &= \kappa \nabla^2 \theta \\
\nabla \cdot (\epsilon \mathbf{E}) &= 0 \\
\epsilon &= \epsilon_2 (1 - e\theta) \\
\mathbf{E} &= -\nabla \phi \\
\rho(\theta) &= \rho (1 - e\theta) \\
\mathbf{g} &= -g \hat{\mathbf{e}}_z \\
\mathbf{g}_e &= \frac{e}{\alpha \rho} \sqrt{\epsilon_2 \mathbf{E}^2} \\
\pi &= \frac{p}{\rho} + gz - \frac{e\theta \epsilon_2 \mathbf{E}^2}{2\rho} - \frac{1}{2} \left( \frac{\partial \epsilon}{\partial \rho} \right)_T \mathbf{E}^2 \\
\theta &= T - T_2
\end{align*}
\]

Control Parameters

\[
\begin{align*}
\eta &= R_1/R_2 = 0.5 \\
\Gamma &= L_z/d = 20 \\
\Delta T &= T_1 - T_2 \\
Gr &= \alpha \Delta T g d^3/\nu^2 = 530 \quad \text{(laminar convective cell)} \\
Pr &= 65 \quad \text{(Silicone oil AK5)}
\end{align*}
\]

\[
V_E : \text{Dimensionless electric potential difference}
\]

\[
V_E = V_0 / \sqrt{\rho \nu \kappa / \epsilon_2} = 0 \sim 10000
\]

\[
g_e = 20.6 \text{ m/s}^2, L = 72306 \quad \text{for } V_E = 10000
\]

\[
L = \alpha g_e \Delta T d^3/\nu \kappa, L \sim V_E^2
\]

\[L : \text{electric Rayleigh number}\]
Numerical methods on CRIANN

- **Numerical method**
  - Finite Volume Method (Cylindrical coordinate system)
  - Fractional Step Method
  - Spatial discretization
    - Central difference scheme
    - QUICK scheme
  - Time advancement
    - 3rd Runge-Kutta scheme
    - 2nd Crank-Nicolson scheme
  - Boundary conditions
    - $u = 0$, $\theta = \Delta T$, $\phi = V_0$ at $r = R_1$
    - $u = 0$, $\theta = 0$, $\phi = 0$ at $r = R_2$
    - $u = 0$, $\partial\theta/\partial z = 0$, $\partial\phi/\partial z = 0$ at $z = 0$, $H$
  - Preconditioned Bi-Conjugate Gradient (PBCG) method (for the electric potential)
  - Boundary conditions
    - $u = 0$, $\theta = \Delta T$, $\phi = V_0$ at $r = R_1$
    - $u = 0$, $\theta = 0$, $\phi = 0$ at $r = R_2$
    - $u = 0$, $\partial\theta/\partial z = 0$, $\partial\phi/\partial z = 0$ at $z = 0$, $H$

- **Computational details**
  - In-house code written by FORTRAN 77 with OpenMP (for parallelization)
  - Storage capacity: 1GB per each computation
  - Computing time: Average 30 days per each computation with 16 cores.
  - CPU base computation
The base flow is maintained up to $V_E = 1100$. $Q = 0.5$
Natural convective cell + DEP buoyancy

- Columnar vortex

\[ V_E = 1,200 \]

\[ V_E = 1,000 \approx 3.85 \text{kV} \]

<table>
<thead>
<tr>
<th>Present (DNS)</th>
<th>Experiment (Meyer et al. 2017)</th>
<th>LSA (Meyer et al. 2017)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{Ec}$</td>
<td>$1100 &lt; V_{Ec} &lt; 1200$</td>
<td>$916 &lt; V_{Ec} &lt; 1099$</td>
</tr>
</tbody>
</table>

\( Q = 0.5 \)

\( z = 10 \)

7 pairs

✓ Stationary columnar vortices
Case 1: Natural convective cell + DEP buoyancy

- Columnar vortex

As $V_E$ increases, vortices gradually grow stronger and longer.
Case 1: Natural convective cell + DEP buoyancy

- Regular wave

\[ V_E = 2500 \]

- A stronger electric body force amplifies the momentum advection in columnar vortices and rises the oscillatory mode. [Busse (JFM, 1972), Clever & Busse (JFM, 1974)]
Case 1: Natural convective cell + DEP buoyancy

- Transition to chaotic flow

- The disturbances of dissymmetric mode arising from regular waves lead to chaos as the electric voltage $V_E$ grows.

- “The symmetry-breaking perturbations in the flow of thermal convection can hasten the chaos by producing a modulation of the rolls.” [McLaughlin & Orszag (JFM, 1982)]
Case 1: Natural convective cell + DEP buoyancy

- Transition to chaotic flow

- Vortices are more branched off into several parts and become more complex for $V_E = 4000$.

- Vortices are split into small ones and the longitudinal ones are no longer dominant for $V_E \geq 6000$.

$V_E = 4000$

$V_E = 6000$

$V_E = 8000$

$Q = 20$

$Q = 60$

$Q = 120$
Variation rate of the kinetic energy \( E_k = \frac{u^2}{2} \)

\[
\frac{dE_k}{dt} = \langle \alpha \theta g u_z \rangle_V - \langle \alpha \theta g_e \cdot u \rangle_V - \langle D \rangle_V
\]

Gravitational buoyancy  Electric buoyancy
Natural convective cell + DEP buoyancy

- Heat transfer rate: Nusselt number

\[ Nu_i = -\frac{\eta \ln \eta}{1 - \eta} \left( \frac{\partial \theta}{\partial r} \right)_{r=R_1} \]

Take-home message: Vortices generated by the electric field cause a significant enhancement of the heat transfer.

Kang & Mutabazi, *J. Appl. Phys.* 125, 184902 (2019), Editor’s pick
Effect of solid-body rotation on DEP-induced convection
DEP + Solid-body Rotation

Rotation transforms stationary helical vortices into oscillatory columnar vortices

\[ Q = 0.002 \]
Conclusion

1. Thermoelectric convection in a cylindrical annular cavity in micro-gravity appears in form of stationary helical vortices.

2. Superimposition of the ground gravity induces a convective cell and leads to stationary columnar, dynamics of which is driven by the electric voltage.

3. The Coriolis force due to solid-body rotation of a cylindrical annulus with $\Delta T$ and $V_E$ transforms helical vortices to columnar vortices.

4. Take-home message: Helical modes of thermoelectric convection in cylindrical annular cavities are destabilized either by solid-body rotation or a small Archimedean buoyancy.

Thermomagnetic convection induced by in ferrofluids

Ferrofluid: colloidal solution of magnetic nanoparticles (e.g. Fe$_2$O$_3$)

Kelvin body force

$$ F_K = M \nabla B = -MB_0 k_b K_1(k_b r) e_r $$

$$ M = M_{ref} (1 - \alpha_m \theta) \quad \theta = T - T_{ref} $$

$$ F_K = \alpha_m M_{ref} B_0 k_b K_1(k_b r) \theta e_r + \nabla [M_{ref} B_0 K_0(k_b r)] $$

$$ g_m = -\frac{\alpha_m M_{ref} B_0 k_b K_1(k_b r)}{\alpha \rho_{ref}} e_r $$
THANK YOU FOR YOUR ATTENTION