Numerical Simulation of thermo-convective flows induced by electric fields in cylindrical annular cavities with dielectric liquids

Changwoo Kang[1] & **Innocent Mutabazi**

Normandie Université, CNRS-Université du Havre,

LOMC UMR 6294, Laboratoire Ondes et Milieux Complexes, Le Havre, France

[1]Department of Mechanical Engineering, Jeonbuk National University

Normandie Université

UNIVERSITÉ LE HAVRE NORMANDIE

LABORATOIRE ONDES et MILIEUX COMPLEXES

cnes CENTRE NATIONAL D'ÉTUDES SPATIALES

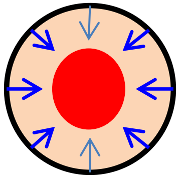
INVESTISSEMENTS D'AVENIR

Labex EMC3 ENERGY MATERIALS & CLEAN COMBUSTION CENTER

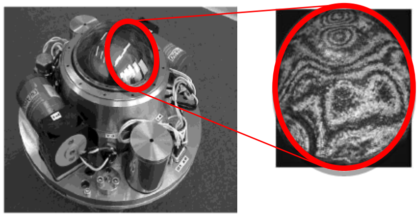
cnrs

LIA n°1092 ISTROF

Introduction



- Geophysics concern : how to make experiment that simulates the convection in planets interior? Need for a real radial gravity
- Hart *et al* (JFM 1986): use of radial dielectrophoretic force in spherical cavity in the Spacelab 3 on the space shuttle Challenger in May 1985
- **ESA project : GeoFlow (2008-2017) :** Experiment in Fluid Science Laboratory of Columbus on ISS by Brandenburg Technology University in Cottbus (LAS, Prof. C. Egbers)



- **ATMOFLOW 2020-2024**

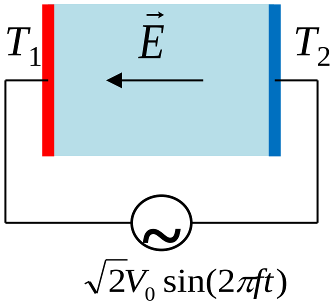
  (DRL for LAS in collaboration with CNES for LOMC)

  Investigation of atmospheric flows near the equatorial zone
- Application 1: Generation of thermal convection in a low-gravity environment
- Application 2 : Enhancement of heat transfer by electric voltage in aerospace (with reduced weight constraints)

ELECTRIC FORCE IN A DIELECTRIC FLUID

- The pondermotive force in a dielectric fluid of density ρ and permittivity ε under the action of an electric field $\vec{E}$ is given by the Helmholtz relation (Landau & Lifshitz, EM, T8)

$$\vec{f} = \underbrace{\rho_e \vec{E}}_{\text{Coulomb force}} \underbrace{- \frac{1}{2}\vec{E}^2\vec{\nabla}\varepsilon}_{\text{Dielectrophoretic (DEP) force}} + \underbrace{\vec{\nabla}\left[\frac{1}{2}\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T \vec{E}^2\right]}_{\text{Electrostriction force}}$$



- High frequency electric field f >> max($1/\tau_\nu$, $1/\tau_\kappa$, $1/\tau_e$, $1/\tau_{ion}$) ⇒

  neglect free charges (Coulomb force)

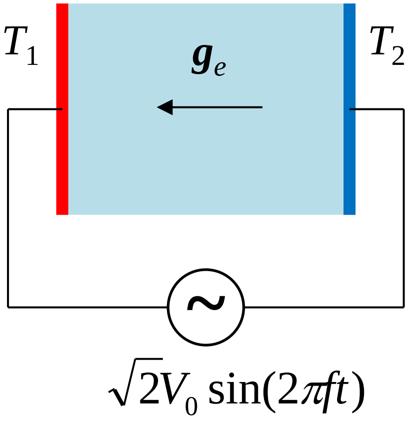
- Electrohydrodynamic Boussinesq approximation : linear variation of the permittivity and density with temperature

$$\rho(T)=\rho_0\left[1-\alpha(T-T_0)\right] \quad \text{and} \quad \varepsilon(T)=\varepsilon_1\left[1-e(T-T_0)\right]$$

ELECTRIC FORCE IN A DIELECTRIC FLUID

- The dielectrophoretic force can be split into a conservative force and buoyancy force

$$\vec{f}_{DEP} = -\frac{1}{2}\vec{E}^{2}\vec{\nabla}\varepsilon = \underbrace{\vec{\nabla}p_{e}}_{\text{conservative force}} + \underbrace{\delta\rho\,\vec{g}_{e}}_{\text{buoyancy force}}$$



- Electric gravity :

$$\vec{g}_{e} = \frac{1}{\rho_0\alpha}\vec{\nabla}\left(\frac{\varepsilon_1 e}{2}\vec{E}^{2}\right)$$

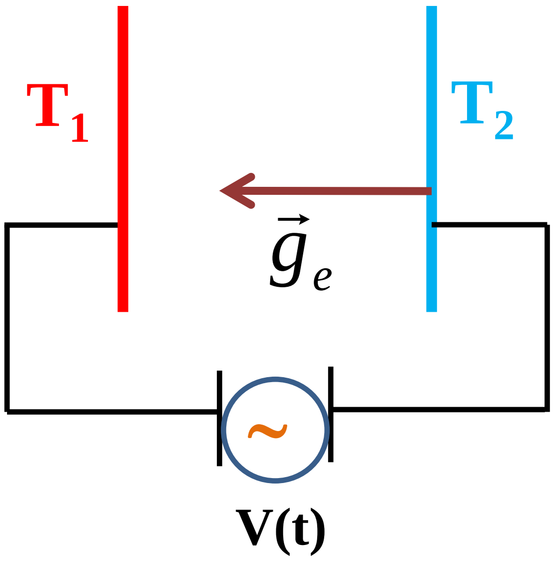
where

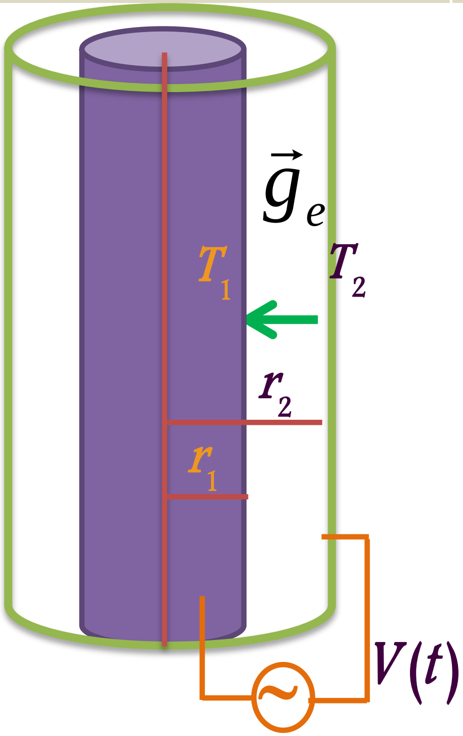
$$\rho(T) = \rho_0\left[1-\alpha(T-T_0)\right]$$
$$\varepsilon(T) = \varepsilon_1\left[1-e(T-T_0)\right]$$

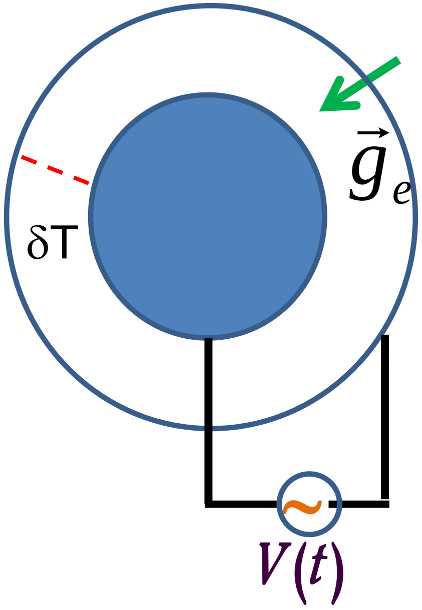
- Magnitude of the electric gravity ~ V^{2}, ε_1, e

ELECTRIC GRAVITY IN SIMPLE GEOMETRIES

Plane Electrode	**Cylindrical Electrode**	**Spherical Electrode**
$g_e \propto \frac{V^2}{d^3}$	$g_e \propto \frac{V^2}{r^3}$	$g_e \propto \frac{V^2}{r^5}$





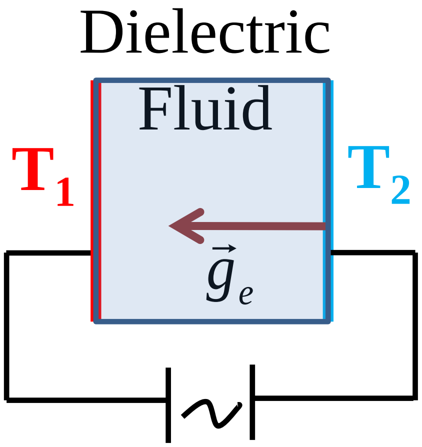


The dielectrophoretic force in spherical geometry models the mechanism of convection in planets [GEOFLOW, B. Futterer & C. Egbers, BTU, Cottbus, Germandy]

Electric gravity for some liquids

The electric gravity in dielectric fluid betwen parallel plates is

$$\boxed{g_e = \frac{\varepsilon_1 e V_0^2}{\alpha \rho_0 d^3}}$$



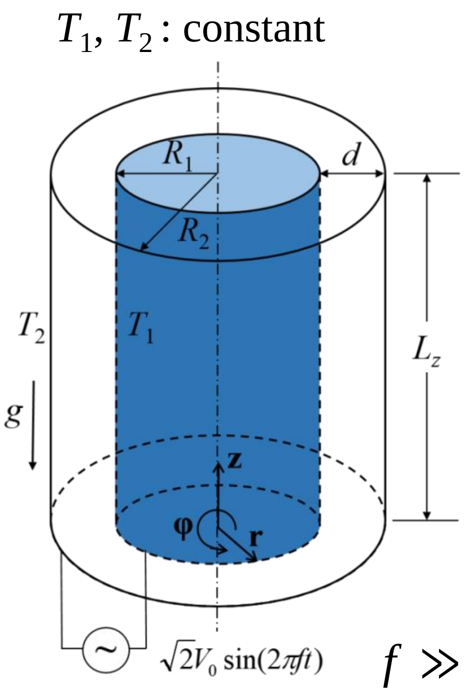
The electric gravity : $g_e \sim V_0^2;\ g_e \sim \dfrac{1}{d^3}$

$V(t) = 2^{1/2} V_0 \sin \omega t$

d = 1cm, ΔT= 1K, V_0 = 1 V

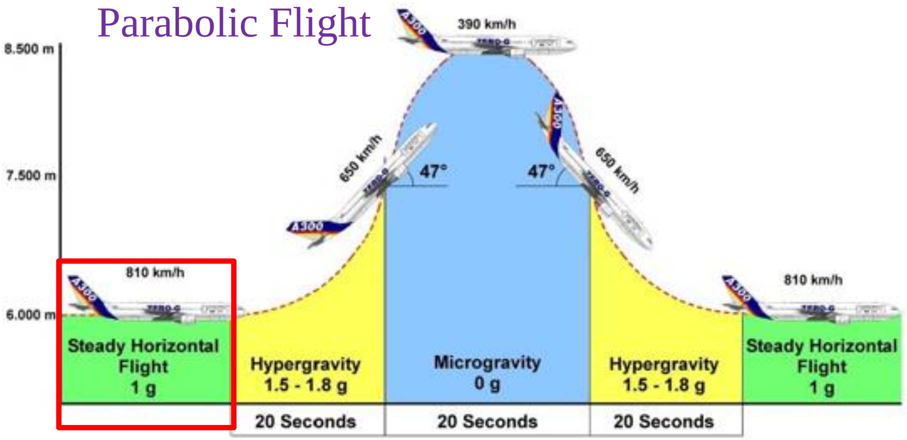
Fluid	$10^{-3}\,\rho$ (kg m^{-3})	$10^3\,\alpha$ (K^{-1})	ϵ_r	e (K^{-1})	g_e (m s^{-2})
Acetonitrile	0.777	1.38	36	0.155	7.11
Nitrobenzene	1.198	0.830	34.9	0.188	10.92
Acetone	0.785	1.43	19.1	0.086	1.11
Chlorobenzene	1.101	0.985	5.61	0.0157	0.01
Silicone oil M5	0.920	1.08	2.7	1.065×10^{-3}	2.73×10^{-5}

Thermoelectric convection in cyindrical annular cavities



$f \gg 1/\tau_\nu,\ 1/\tau_\kappa$

Flow geometry



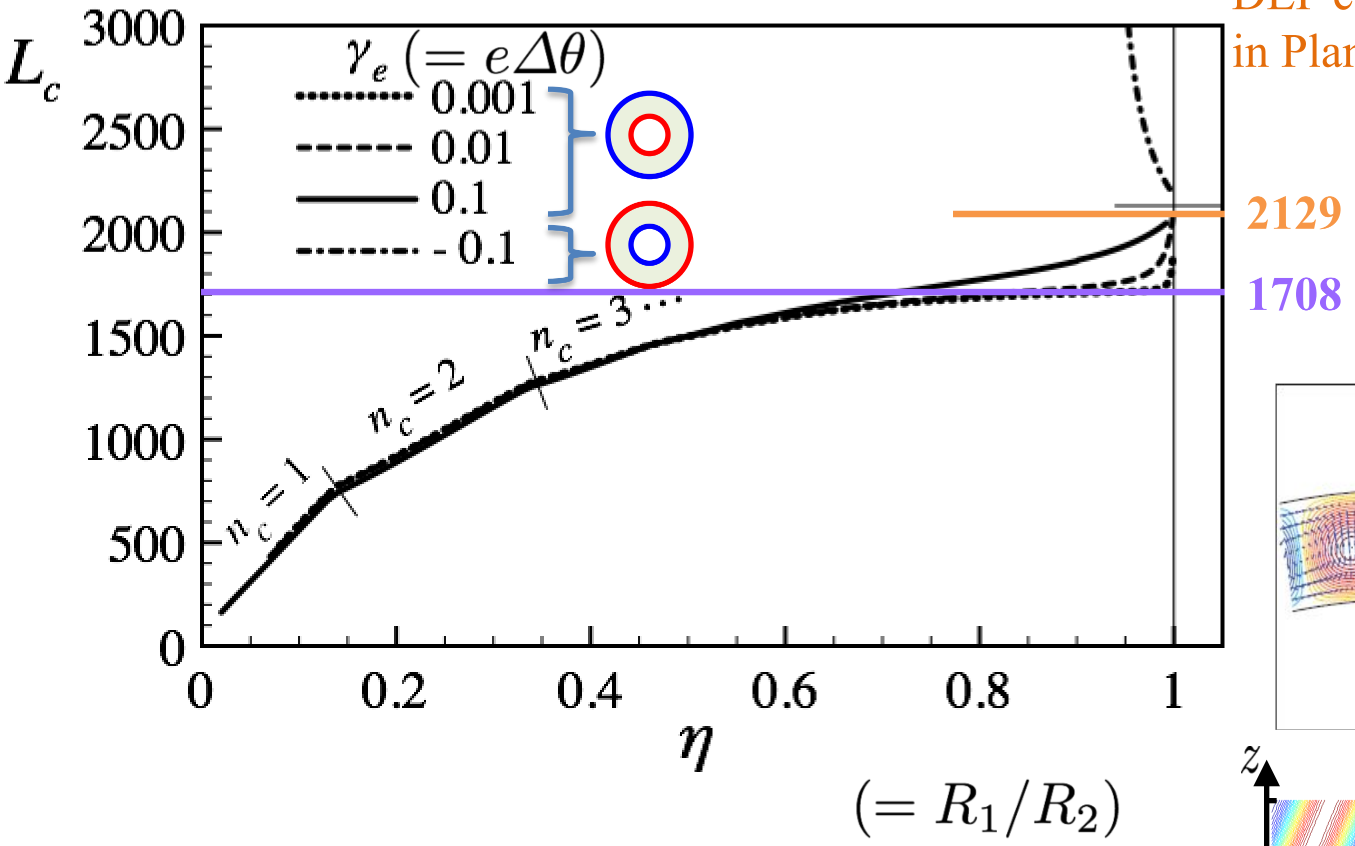
1*g*

Experiments in parabolic flight campaigns funded by CNES & DLR

✓ The effect of a thermo-electric body force on the flow of a dielectric liquid with a radial temperature gradient and an alternating electric voltage in cylindrical annular cavities has been studied

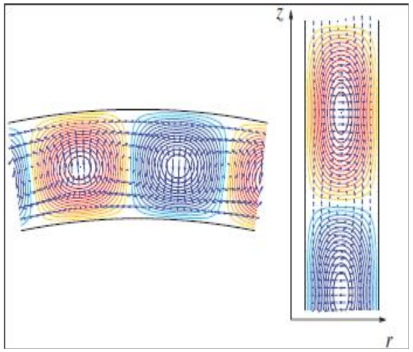
- by linear stability analysis (infinitely long annulus)
- by weakly nonlinear analysis (infinitely long annulus)
- by direct numerical simulations (DNS) for $\eta = R_1/R_2 = 0.5$, $\Gamma = L_z/d = 20$

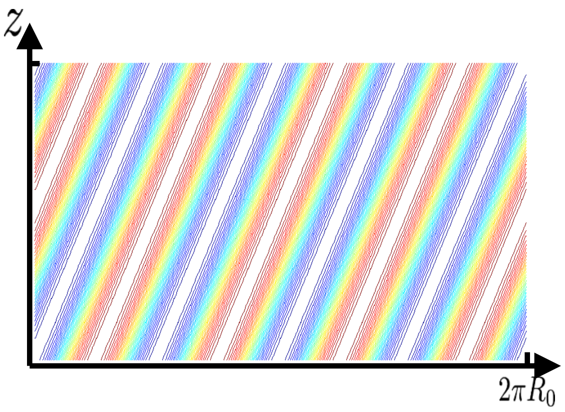
LSA in a cylindrical annulus: Critical electric Rayleigh number



DEP convection
in Plane Geometry

Rayleigh-Bénard
convection



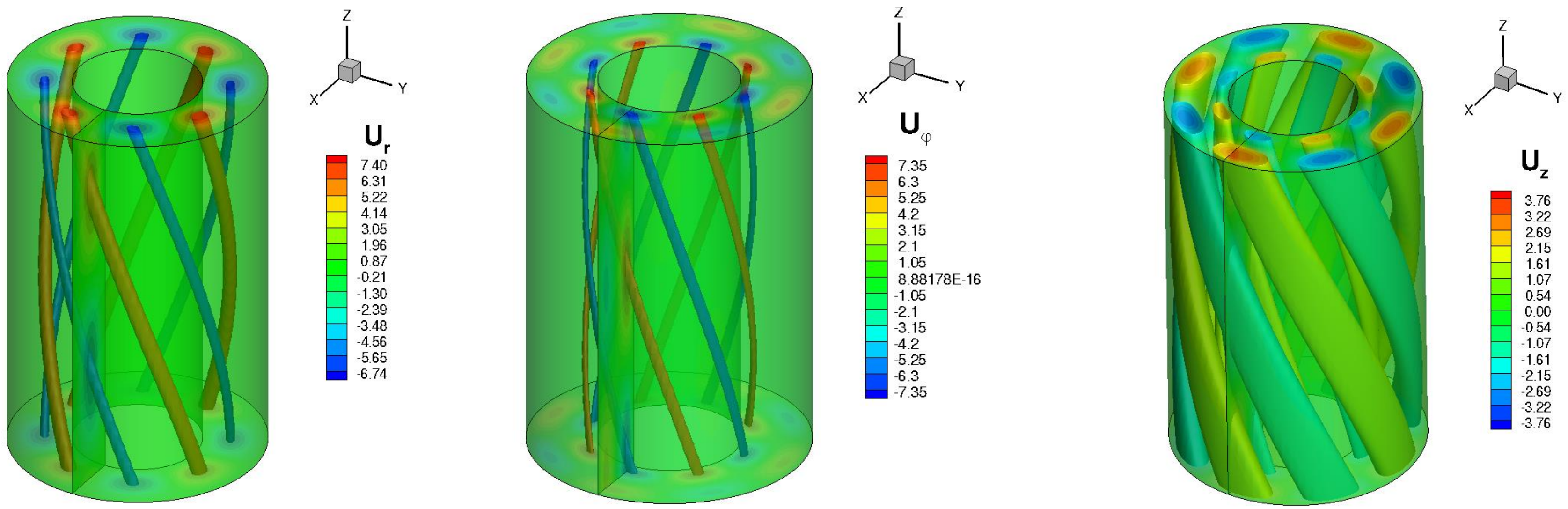


L_c

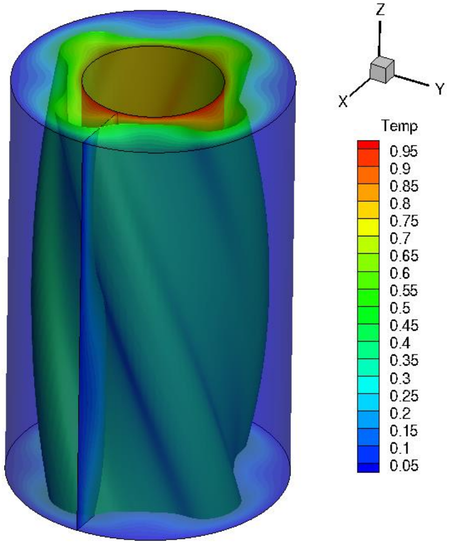
- is independent on Pr;
- depends sensitively on η and on γ_e

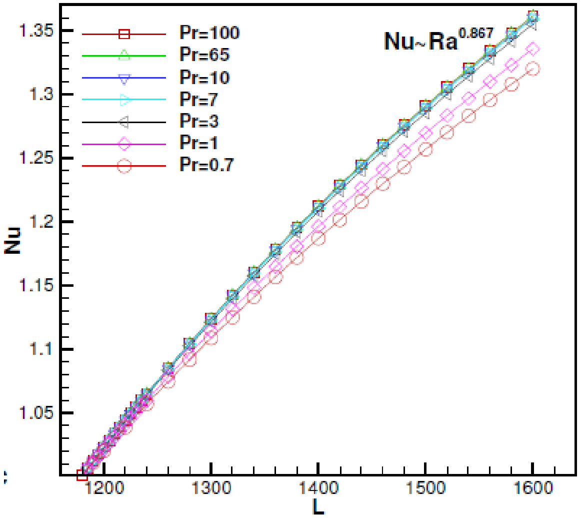
(Yoshikawa *et al. Phys. Fluids* **25**, 024106, 2013)

Heat transfer in the thermoelectric convection in microgravity

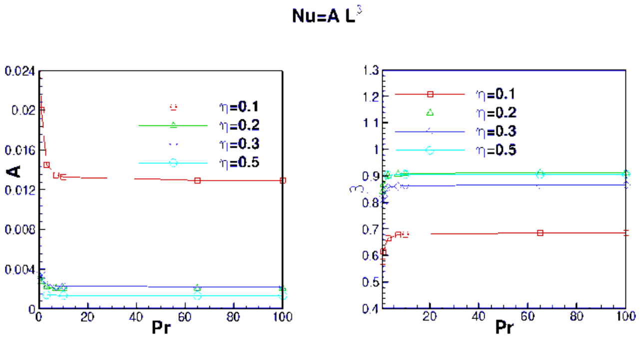


The 3 velocity components are of the same order of magnitude in contrast with the Couette-Taylor problem where the azimuthal component is dominant



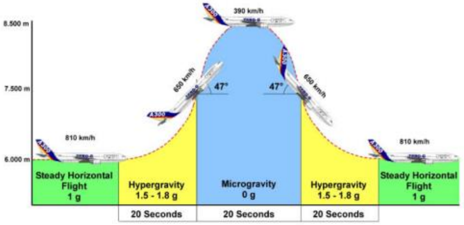


$$\boxed{Nu(L;\Pr,\eta)=A(\Pr,\eta)L^{\beta(\Pr,\eta)}}$$

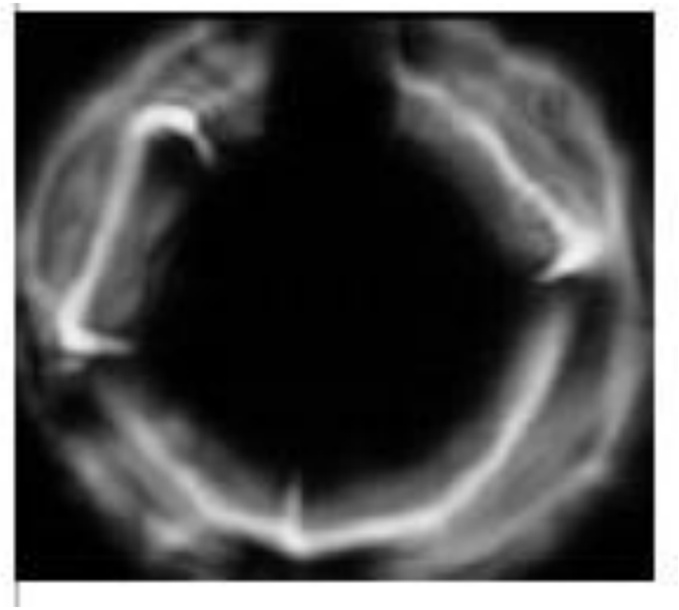


Travnikov *et al.*, *Phys. Fluids* **27** (2015)

Experiment in parabolic flight

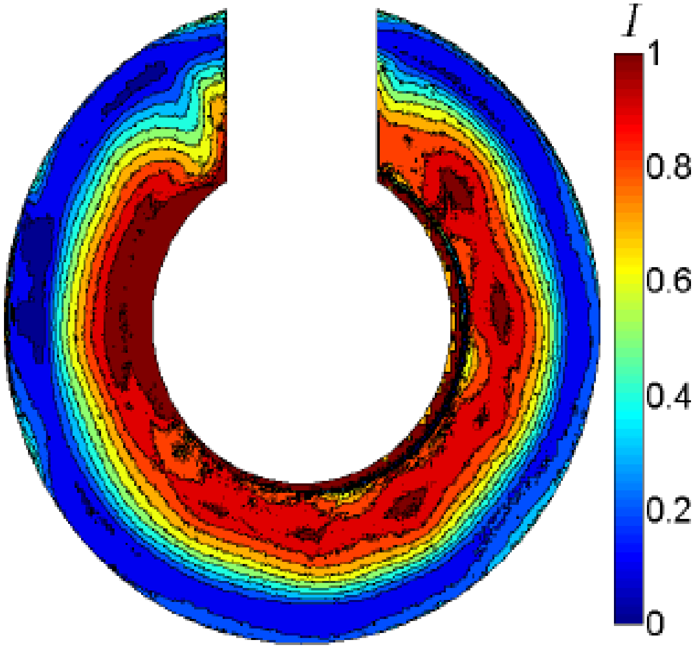


Sitte, 2000



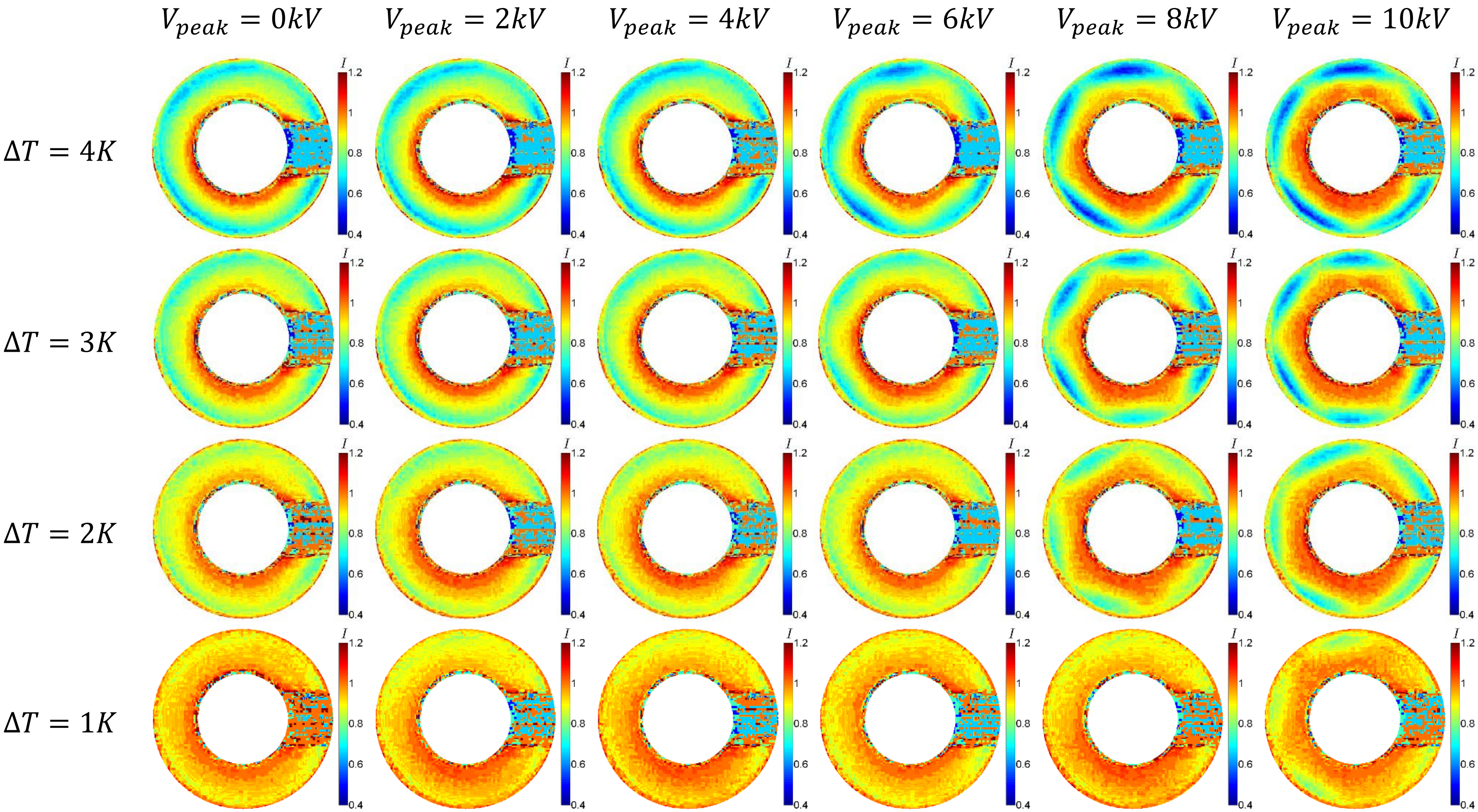
L = 17600

Meyer *et al*. 2016

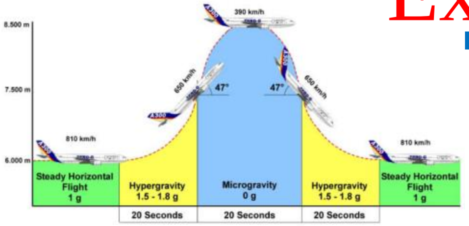


Do we observe columnar or helical vortices?

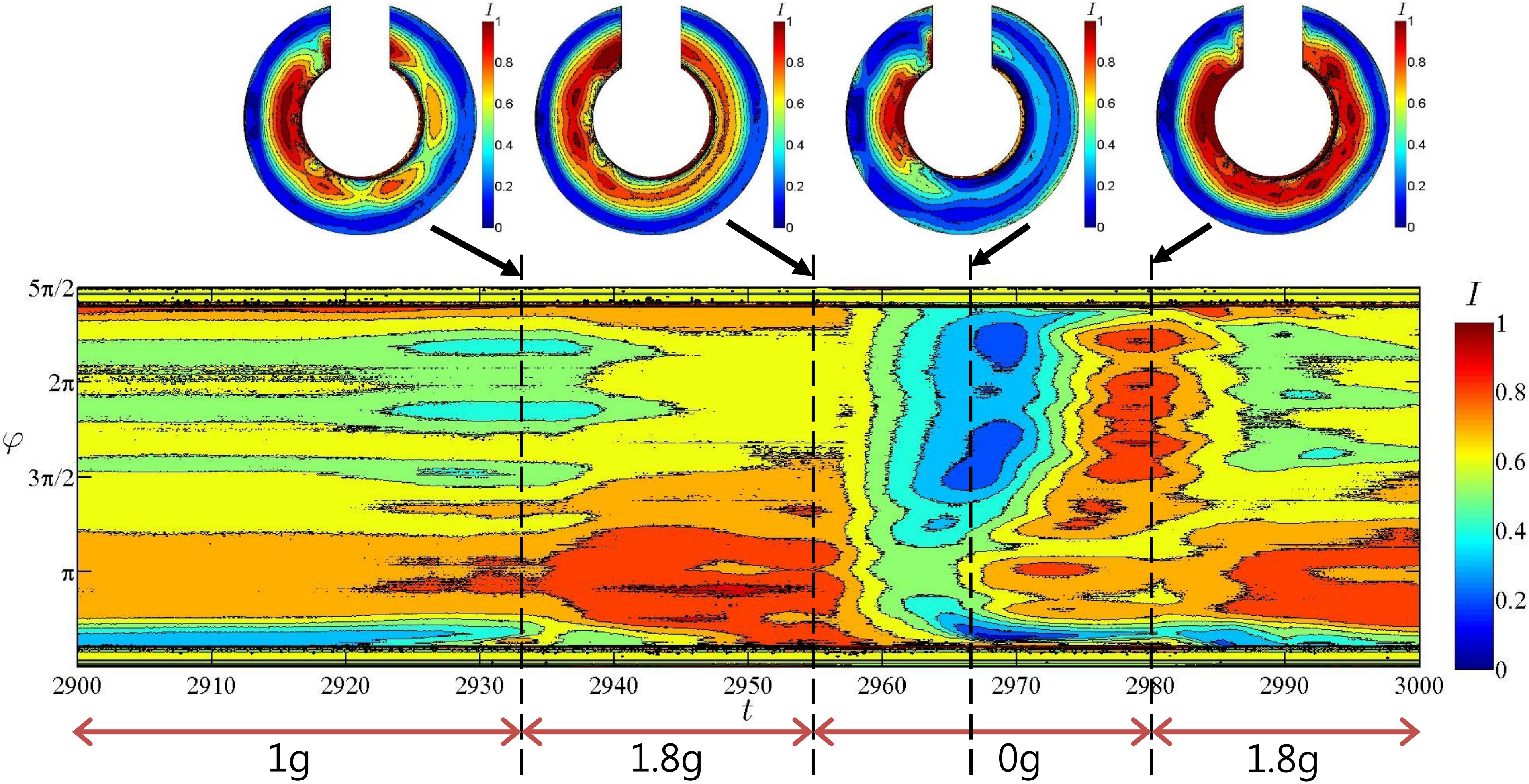
Experiment in parabolic flight



Experiment in parabolic flight



Instantaneous shadowgraph pictures



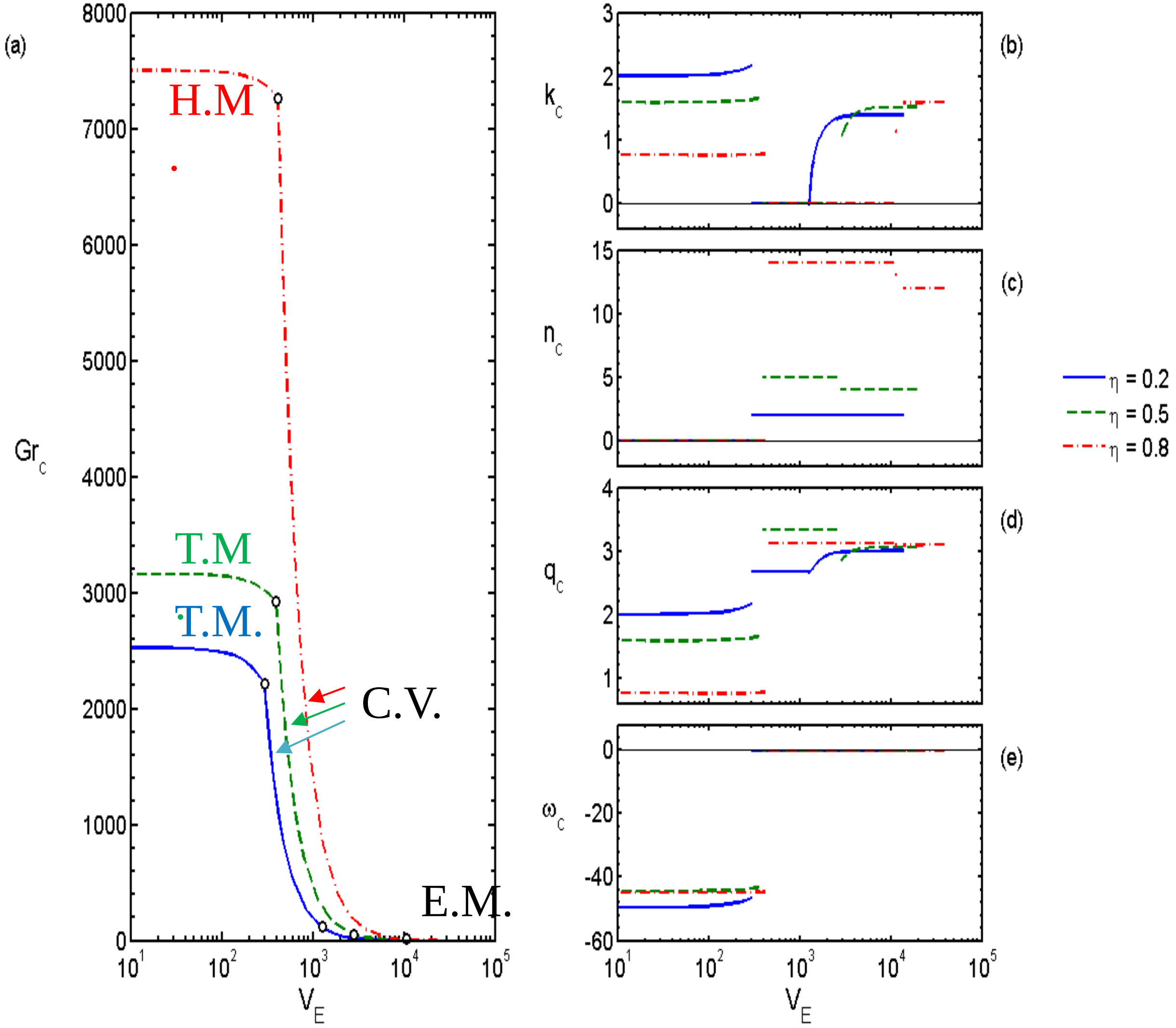
$\Delta T = 10K$
$V_E = 3298$

Spatio-temporal diagram (t, φ) of the intensity at the median surface

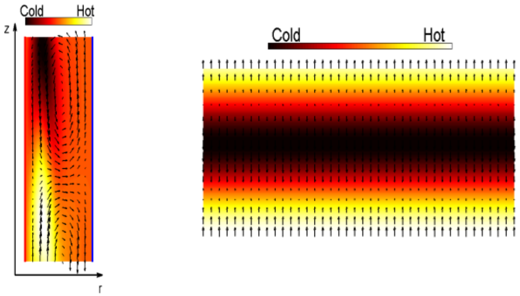
Natural convective cell + DEP buoyancy

Critical parameters for Pr = 10

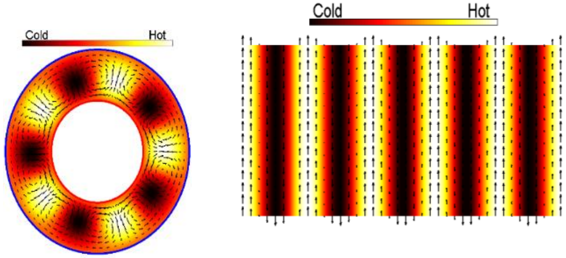
Ga = 1370 ; δ = 0.1



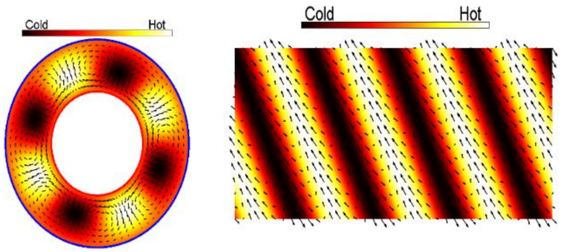
Thermal Mode



Column Mode



Electric Mode



Meyer et al. CRME 345 (2017)

Problem Formulation

- Governing equations (Electro-hydrodynamic Boussinesq approximation)

$$\nabla \cdot \mathbf{u} = 0$$

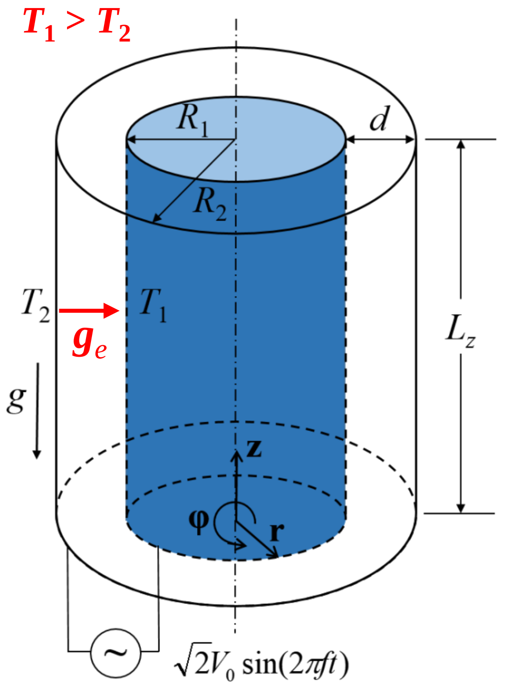
$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla \pi + \nu \nabla^2 \mathbf{u} - \alpha\theta(\mathbf{g} + \mathbf{g}_e)$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{u} \cdot \nabla)\theta = \kappa \nabla^2 \theta$$

$$\nabla \cdot (\epsilon \boldsymbol{E}) = 0 \qquad \epsilon = \epsilon_2(1 - e\theta) \qquad \boldsymbol{E} = -\nabla \phi$$

$$\rho(\theta) = \rho(1 - e\theta) \qquad \mathbf{g} = -g\vec{\boldsymbol{e}}_z \qquad \mathbf{g}_e = \frac{e}{\alpha\rho}\nabla\frac{\epsilon_2 \boldsymbol{E}^2}{2}$$

$$\pi = \frac{p}{\rho} + gz - \frac{e\theta\epsilon_2 \boldsymbol{E}^2}{2\rho} - \frac{1}{2}\left(\frac{\partial \epsilon}{\partial \rho}\right)_T \boldsymbol{E}^2 \qquad \theta = T - T_2$$



< Schematic of flow geometry >

✓ Control Parameters

$\eta = R_1/R_2 = 0.5$

$\Gamma = L_z/d = 2\ 0$ $\quad \Delta T = T_1 - T_2$

$Gr = \alpha\Delta T g d^3/\nu^2 = 530$ (laminar convective cell)

$Pr = 65$ (Silicone oil AK5)

V_E : Dimensionless electric potential difference

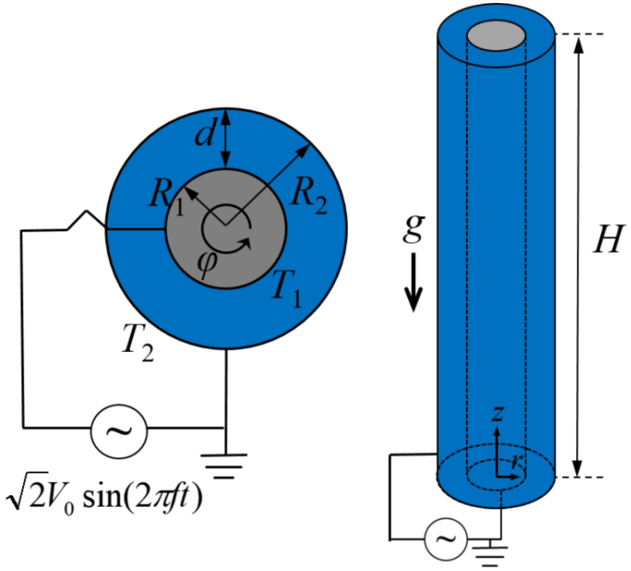
$$\boxed{V_E = V_0/\sqrt{\rho\nu\kappa/\epsilon_2} = 0 \sim 10\ 000}$$

$g_e = 20.6$ m/s^2, $L = 72\ 306$ for $V_E = 10\ 000$

$L = \alpha g_e \Delta T d^3/\nu\kappa$, $L \sim V_E^2$

L : electric Rayleigh number

Numerical methods on CRIANN



➢ Numerical method

- ✓ Finite Volume Method (Cylindrical coordinate system)
- ✓ Fractional Step Method
  - ✓ Spatial discretization
    - - Central difference scheme
    - - QUICK scheme
  - ✓ Time advancement
    - - 3rd Runge-Kutta scheme
    - - 2nd Crank-Nicolson scheme
- ✓ Preconditioned Bi-Conjugate Gradient (PBCG) method (for the electric potential)
  - ✓ Boundary conditions

$$\mathbf{u} = 0,\ \theta = \Delta T,\ \phi = V_0 \quad \text{at } r = R_1$$

$$\mathbf{u} = 0,\ \theta = 0,\ \phi = 0 \quad \text{at } r = R_2$$

$$\mathbf{u} = 0,\ \partial\theta/\partial z = 0,\ \partial\phi/\partial z = 0 \quad \text{at } z = 0, H$$

- ✓ Grid resolution : $64(r) \times 128(\varphi) \times 256(z)$

➢ Computational details

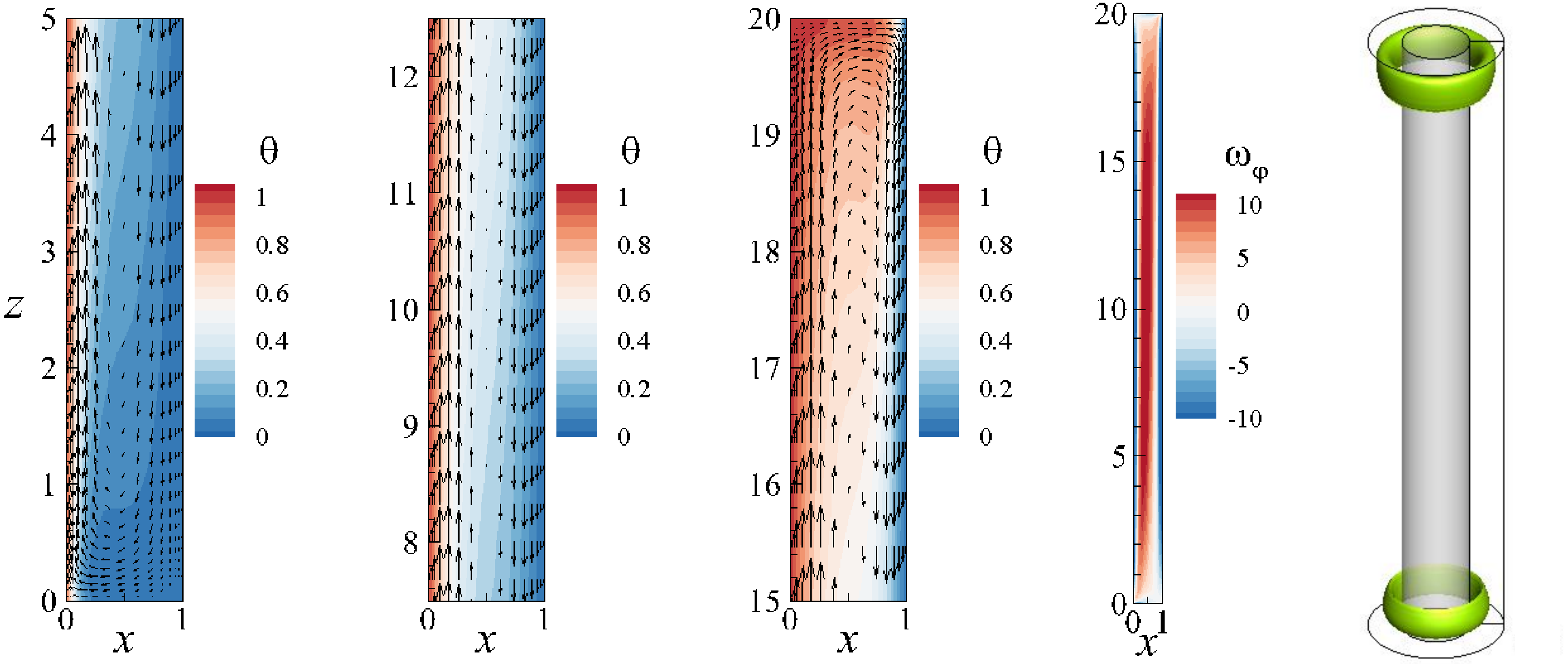
- ✓ In-house code written by FORTRAN 77 with OpenMP (for parallelization)
- ✓ Storage capacity : 1GB per each computation
- ✓ Computing time : Average 30 days per each computation with 16 cores.
- ✓ CPU base computation

Natural convective cell + DEP buoyancy

- Base flow

$V_E = 0$

$Q = 0.5$



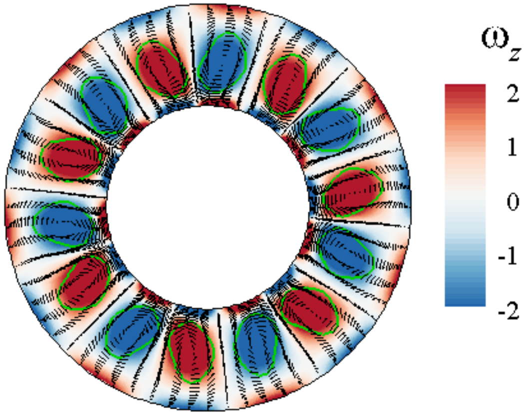
✓ The base flow is maintained up to V_E =1100.

Natural convective cell + DEP buoyancy

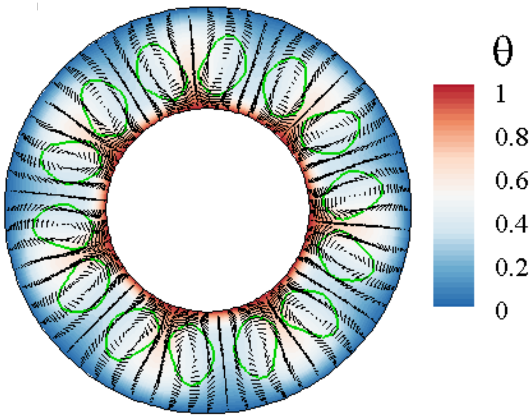
- Columnar vortex

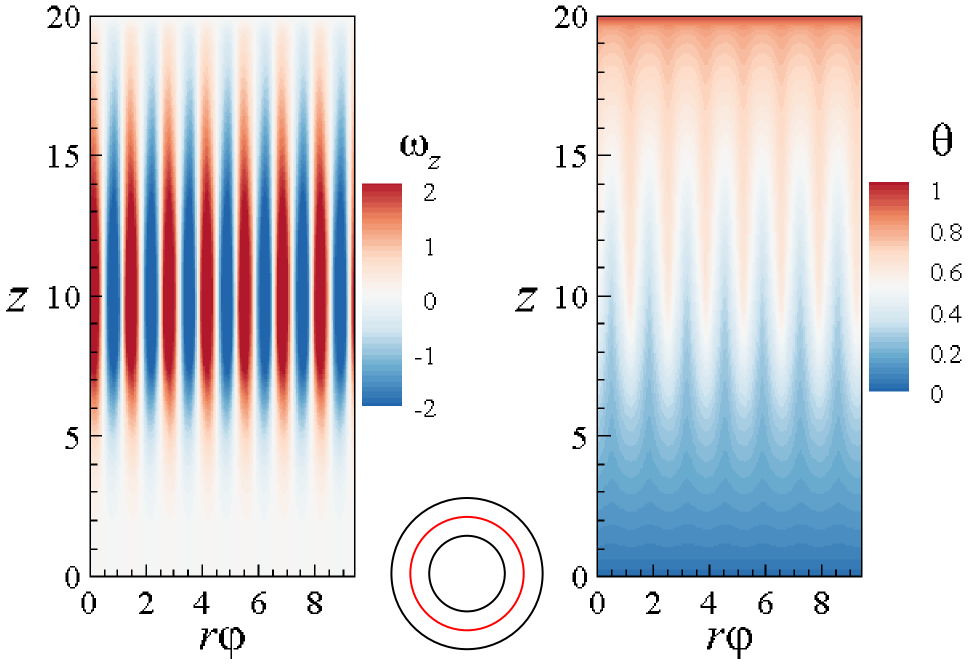
$V_E = 1{,}200$

$z = 10$



7 pairs





$Q = 0.5$

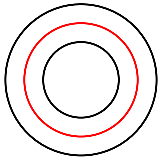
✓ Stationary columnar vortices

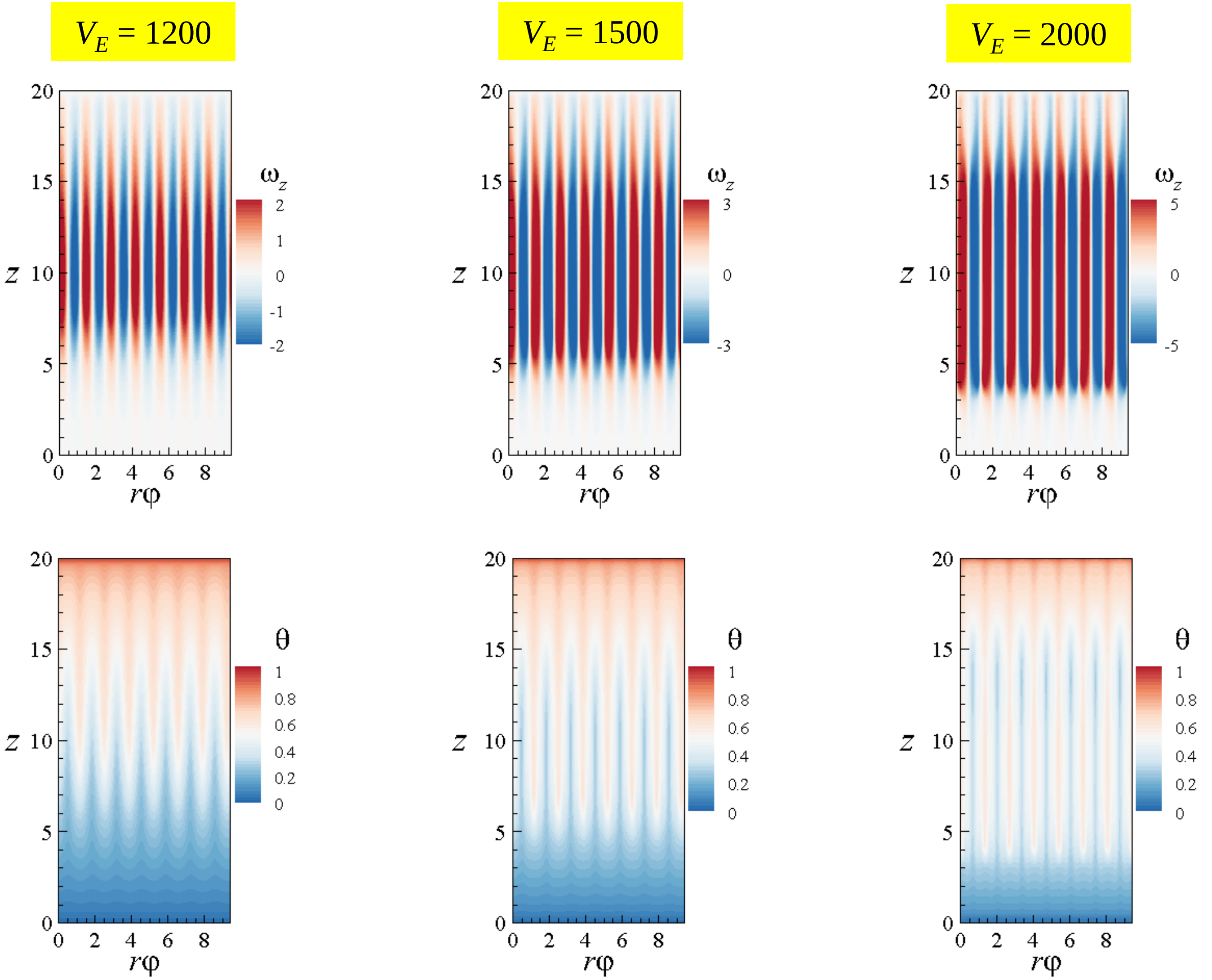
$V_E = 1{,}000 \approx 3.85$ kV

	Present (DNS)	Experiment (Meyer *et al*. 2017)	LSA (*Meyer et al*. 2017)
V_{E_c}	$1100 < V_{E_c} < 1200$	$916 < V_{E_c} < 1099$	479

Case 1 : Natural convective cell + DEP buoyancy

- Columnar vortex



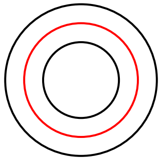


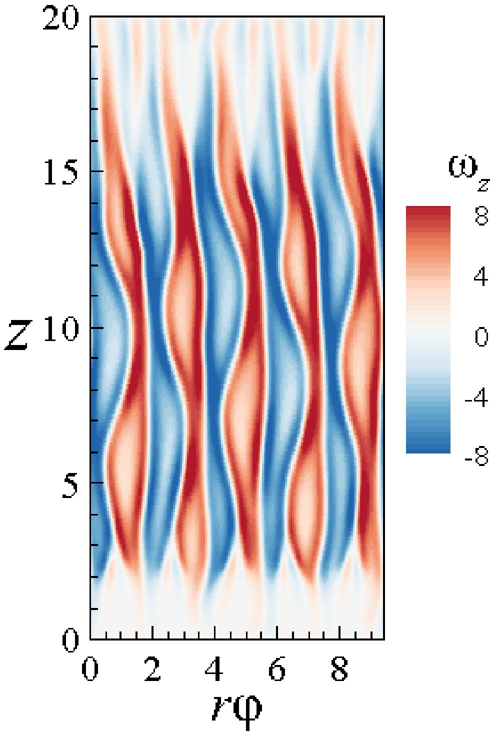
✓ As V_E increases, vortices gradually grow stronger and longer.

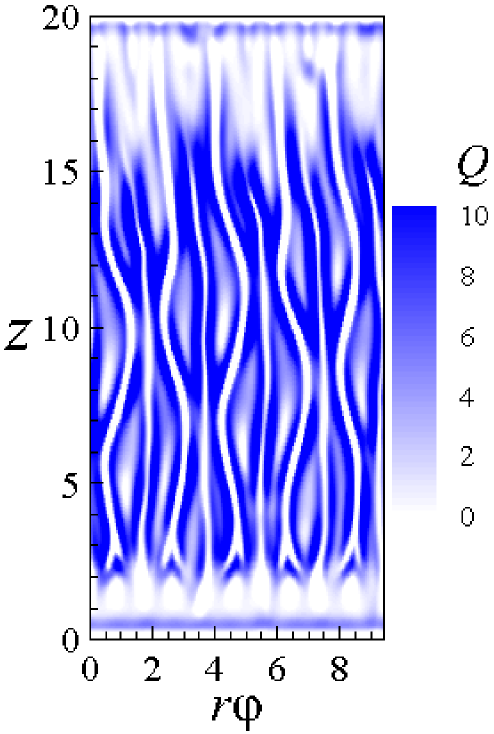
Case 1 : Natural convective cell + DEP buoyancy

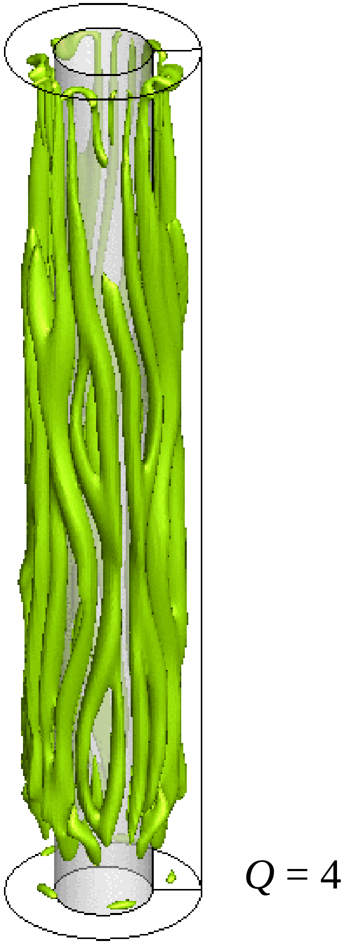
- Regular wave

$V_E = 2500$







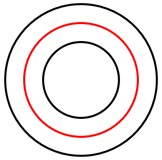


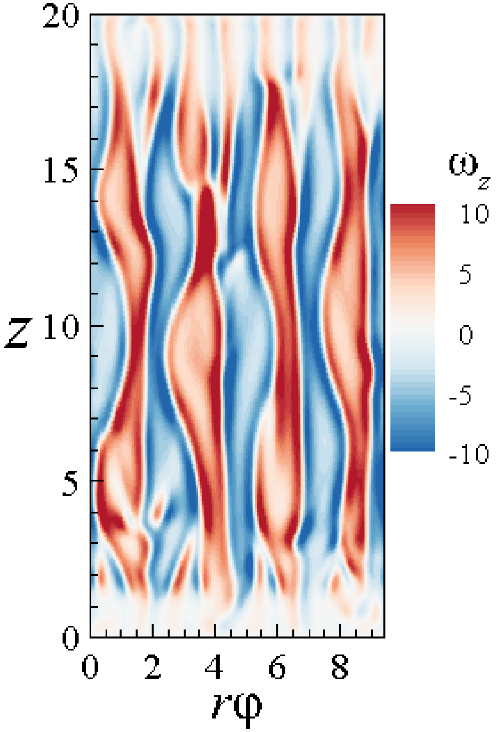
- ✓ A stronger electric body force amplifies the momentum advection in columnar vortices and rises the oscillatory mode. [Busse (JFM, 1972), Clever & Busse (JFM, 1974)]

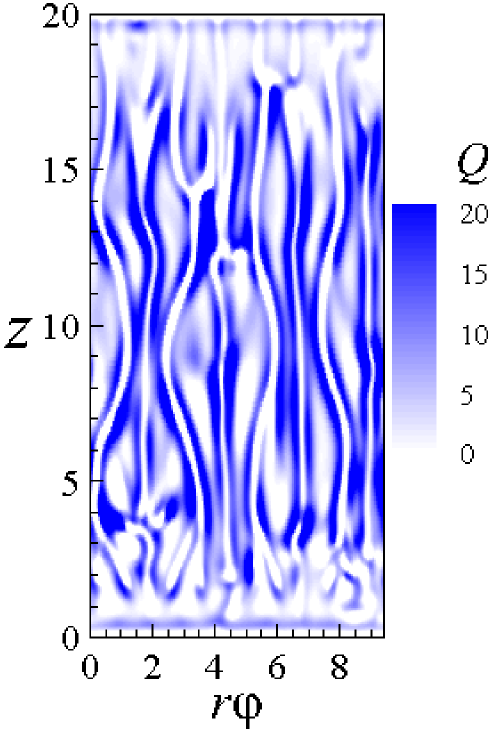
Case 1 : Natural convective cell + DEP buoyancy

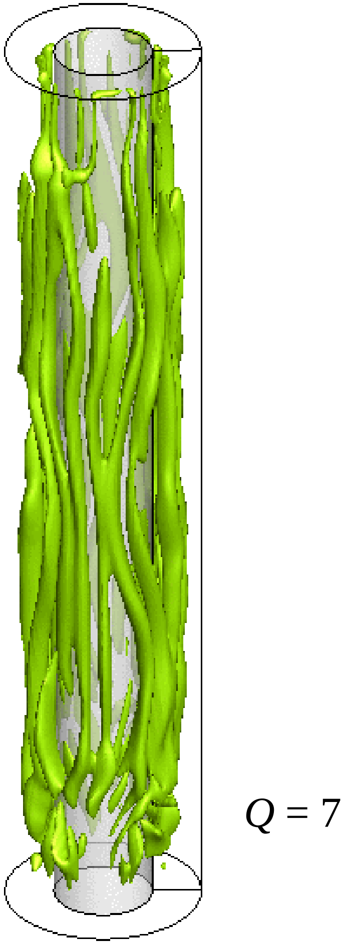
- Transition to chaotic flow

$V_E = 3000$







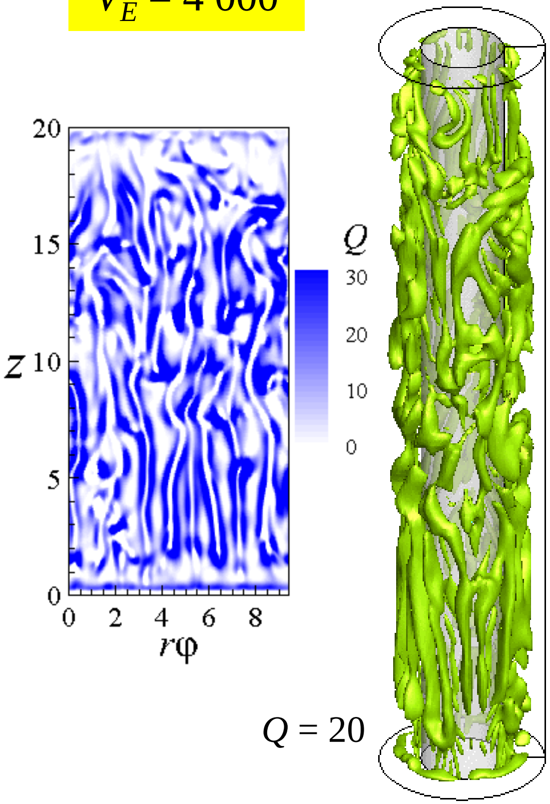


- ✓ "The symmetry-breaking perturbations in the flow of thermal convection can hasten the chaos by producing a modulation of the rolls." [McLaughlin & Orszag (JFM, 1982)]
- ✓ The disturbances of dissymmetric mode arising from regular waves lead to chaos as the electric voltage V_E grows.

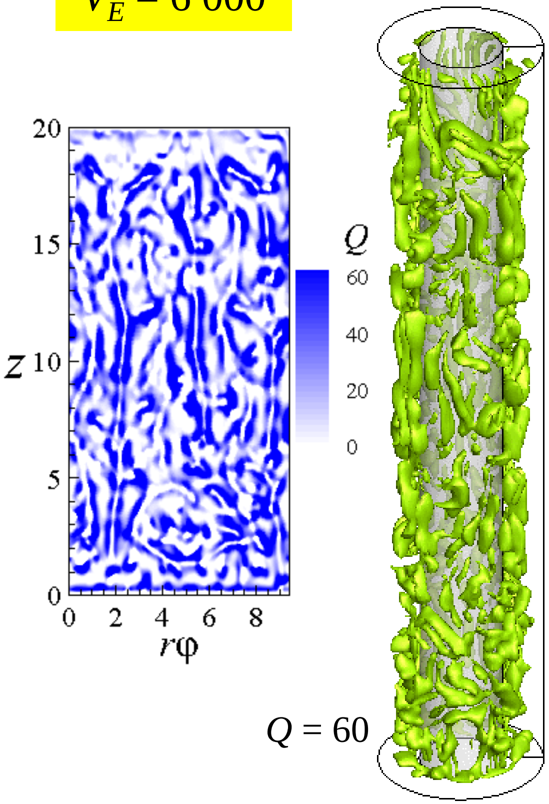
Case 1 : Natural convective cell + DEP buoyancy

- Transition to chaotic flow

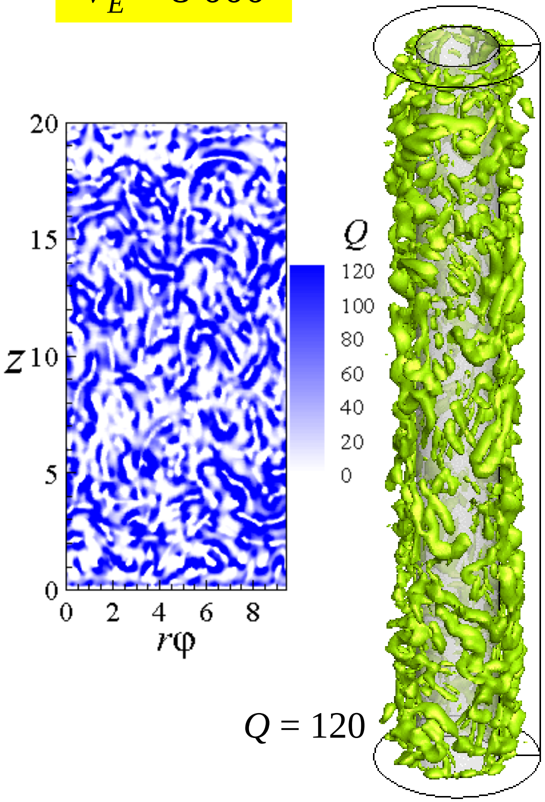
$V_E = 4\ 000$



$V_E = 6\ 000$



$V_E = 8\ 000$



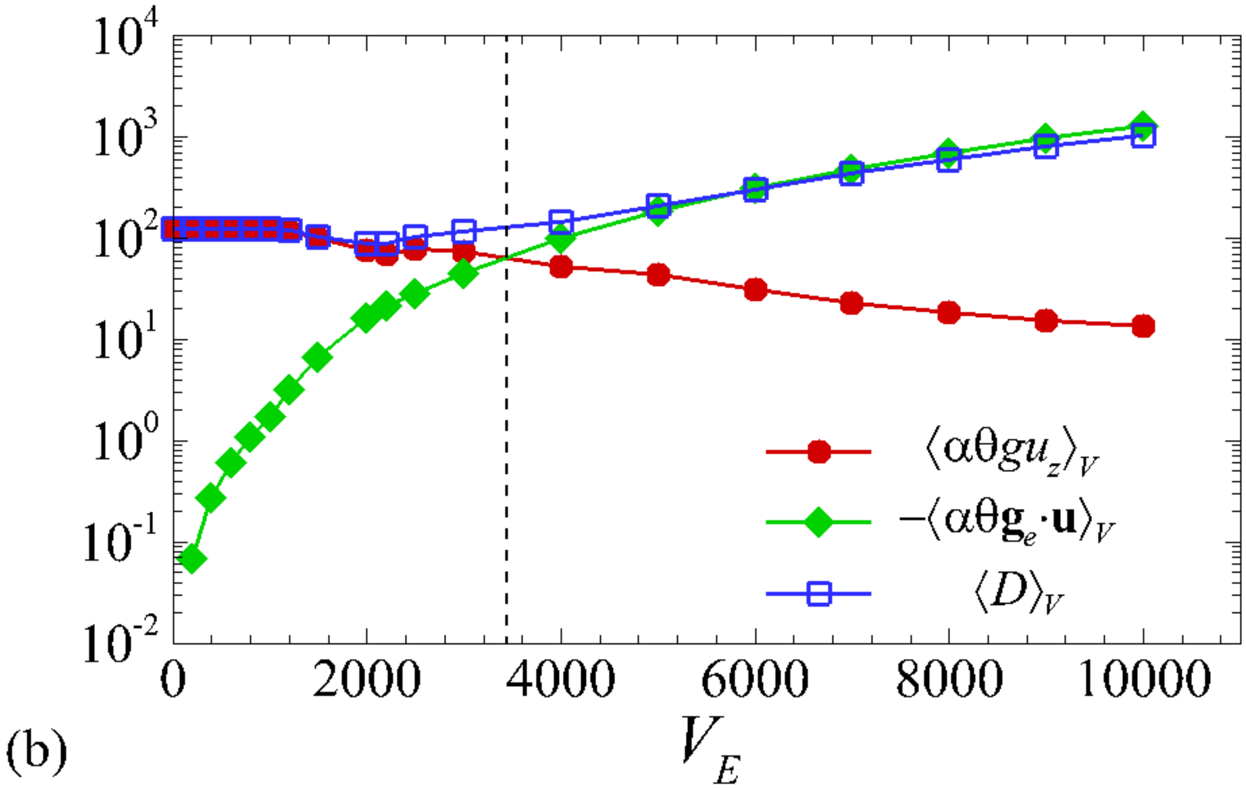
- ✓ Vortices are more branched off into several parts and become more complex for $V_E = 4\ 000$.
- ✓ Vortices are split into small ones and the longitudinal ones are no longer dominant for $V_E \geq 6\ 000$.

Natural convective cell + DEP buoyancy

- Variation rate of the kinetic energy $E_k = \mathbf{u}^2/2$

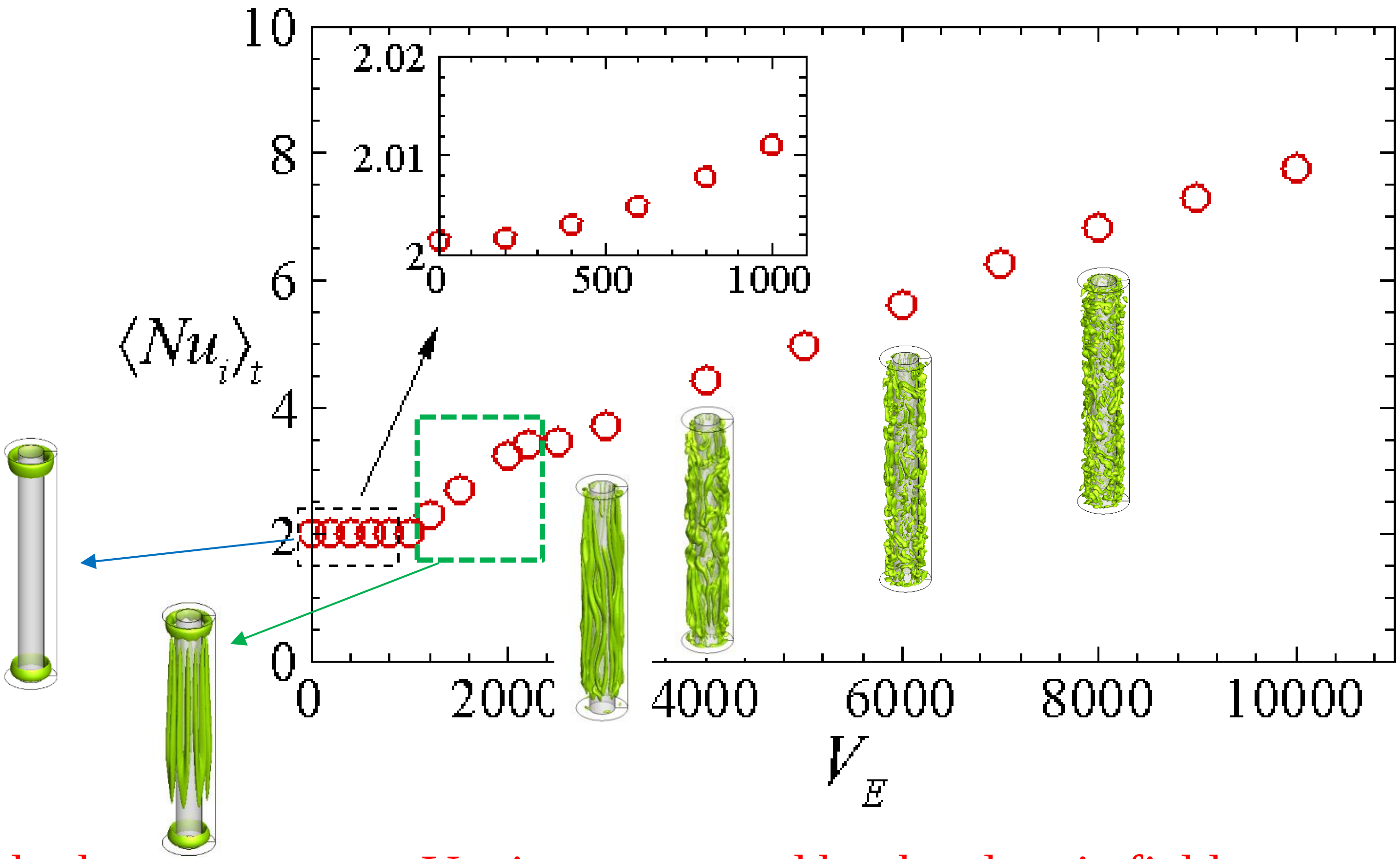
Viscous energy dissipation

$$\frac{dE_k}{dt} = \underbrace{\langle \alpha\theta g u_z \rangle_V}_{\text{Gravitational buoyancy}} - \underbrace{\langle \alpha\theta \mathbf{g}_e \cdot \mathbf{u} \rangle_V}_{\text{Electric buoyancy}} - \langle D \rangle_V$$



Natural convective cell + DEP buoyancy

- Heat transfer rate : Nusselt number $Nu_i = -\frac{\eta \ln \eta}{1-\eta}\left(\frac{\partial\theta}{\partial r}\right)_{r=R_1}$

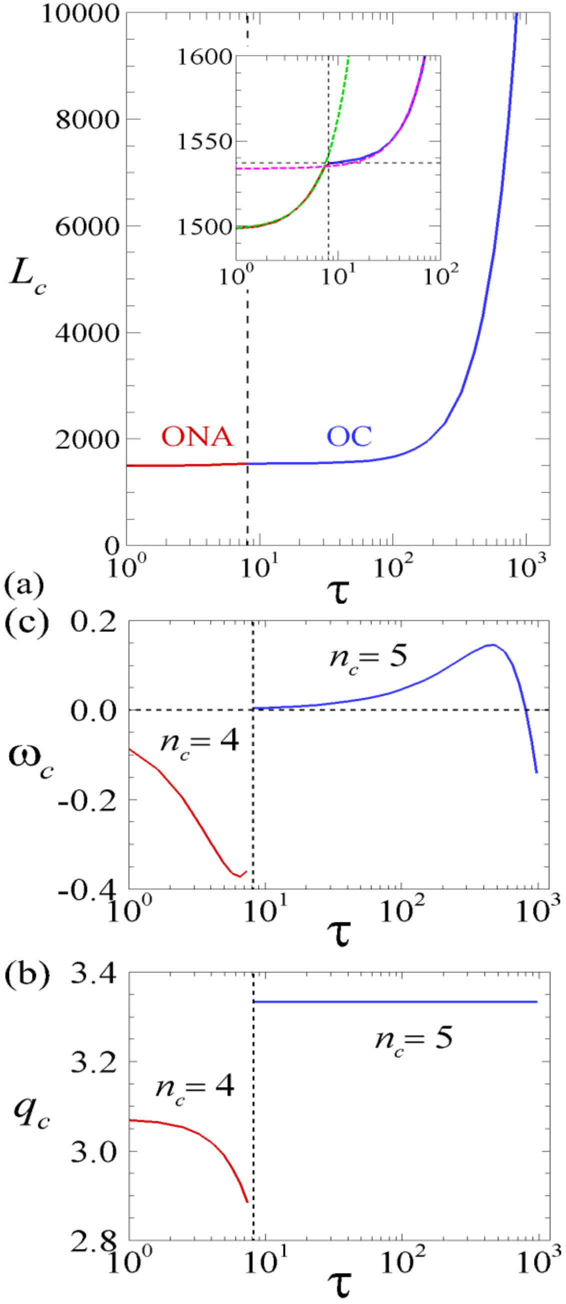


Take-home message : Vortices generated by the electric field cause a significant enhancement of the heat transfer.

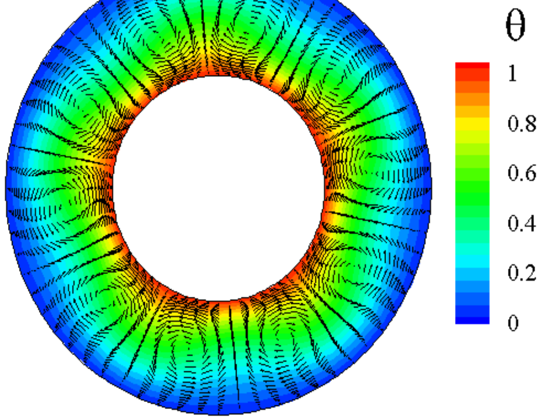
Kang & Mutabazi, *J. Appl. Phys.* **125**, 184902 (2019), Editor's pick
Kang & Mutabazi, *J. Fluid. Mech.* **908**, A26 (2021),

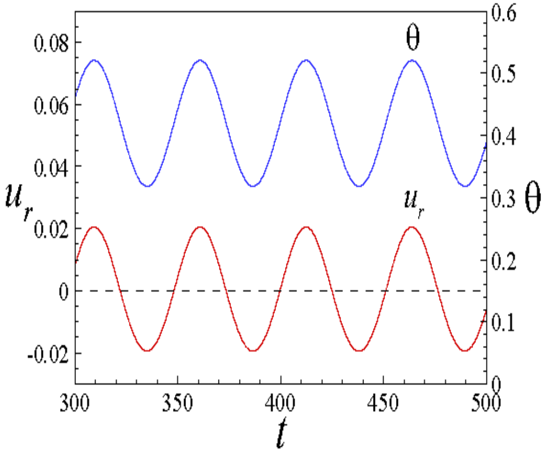
Effect of solid-body rotation on DEP-induced convection

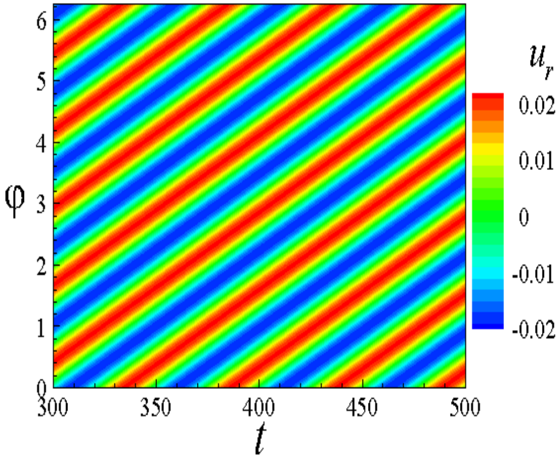
DEP + Solid-body Rotation



$Q = 0.002$







Rotation transforms stationary helical vortices into oscillatory columnar vortices

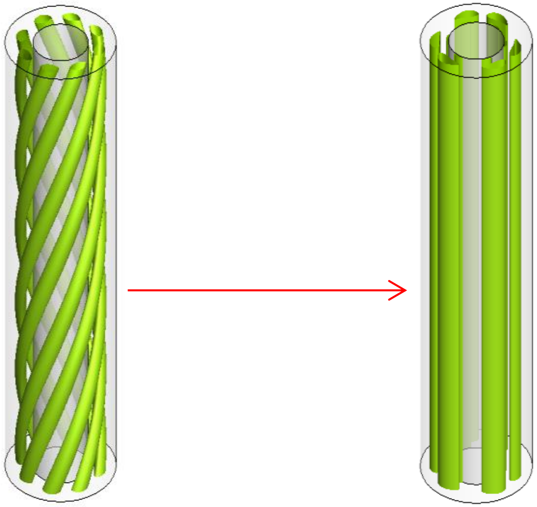
Conclusion

1. Thermoelectric convection in a cylindrical annular cavity in micro-gravity appears in form of stationary helical vortices.

2. Superimposition of the ground gravity induces a convective cell and leads to stationary columnar, dynamics of which is driven by the electric voltage.

3. The Coriolis force due to solid-body rotation of a cylindrical annulus with ΔT and V_E transforms helical vortices to columnar vortices

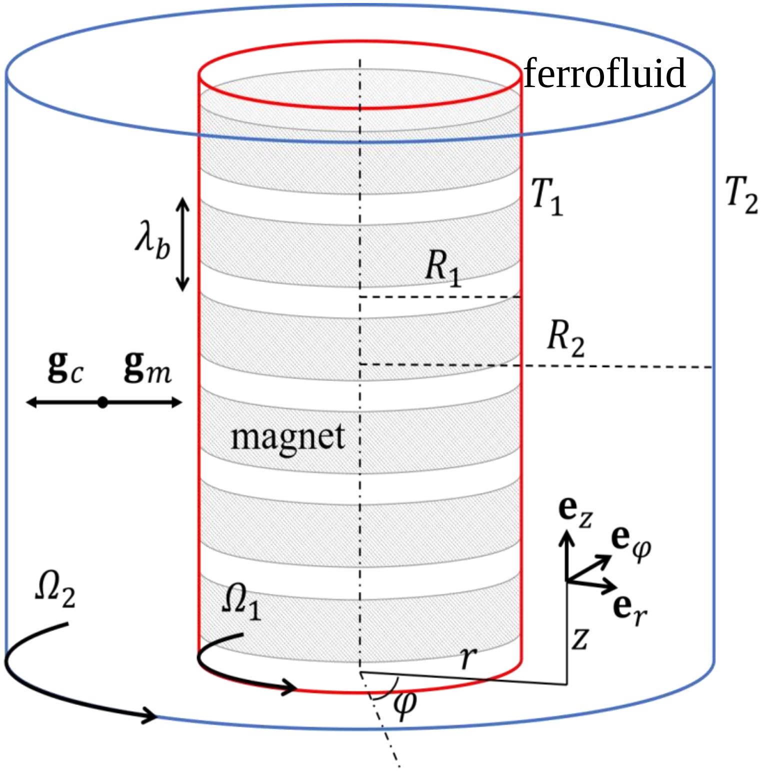
4. Take-home message : Helical modes of thermoelectric convection in cylindrical annular cavities are destabilized either by solid-body rotation or a small Archimedean buoyancy



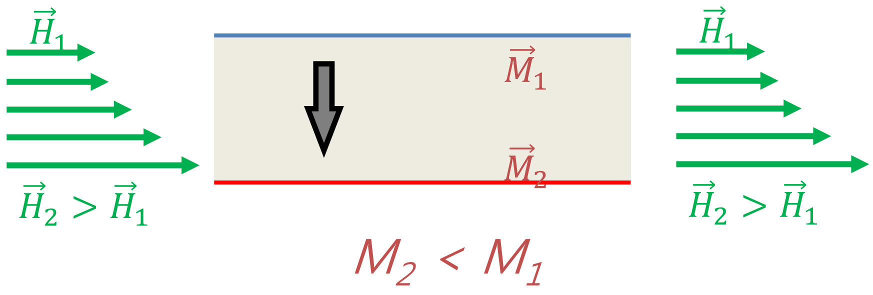
Kang, Meyer, Yoshikawa & Mutabazi, *Phys. Rev. Fluids* **4**, 093502 (2019)

OUTLOOK

Thermomagnetic convection induced by in ferrofluids



Ferrofluid :
colloidal solution of magnetic nanoparticles (e.g. Fe_2O_3)



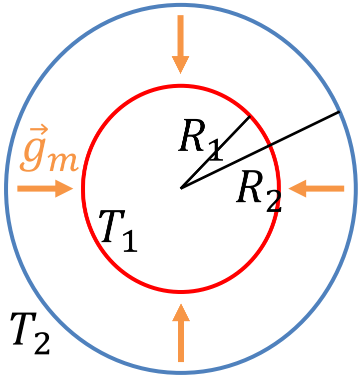
Kelvin body force

$$\boldsymbol{F}_K = M\boldsymbol{\nabla} B = -MB_0k_bK_1(k_br)\boldsymbol{e}_r$$

$$M = M_{ref}\left(1-\alpha_m\theta\right) \quad \theta = T - T_{ref}$$

$$\boldsymbol{F}_K = \underbrace{\alpha_m M_{ref} B_0 k_b K_1(k_b r)\theta \boldsymbol{e}_r}_{-\alpha\theta\rho_{ref}\boldsymbol{g}_m} + \boldsymbol{\nabla}\left[M_{ref}B_0K_0(k_br)\right]$$

$$\boldsymbol{g}_m = -\frac{\alpha_m M_{ref} B_0 k_b K_1(k_b r)}{\alpha\rho_{ref}}\boldsymbol{e}_r$$



THANK YOU FOR YOUR ATTENTION

Liberté • Égalité • Fraternité
RÉPUBLIQUE FRANÇAISE
MINISTÈRE
DE L'ENSEIGNEMENT SUPÉRIEUR
ET DE LA RECHERCHE

RÉGION NORMANDIE

cnes
CENTRE NATIONAL D'ÉTUDES SPATIALES

cnrs

LIA n°1092 ISTROF

INVESTISSEMENTS D'AVENIR

LabexEMC3
ENERGY MATERIALS & CLEAN COMBUSTION CENTER